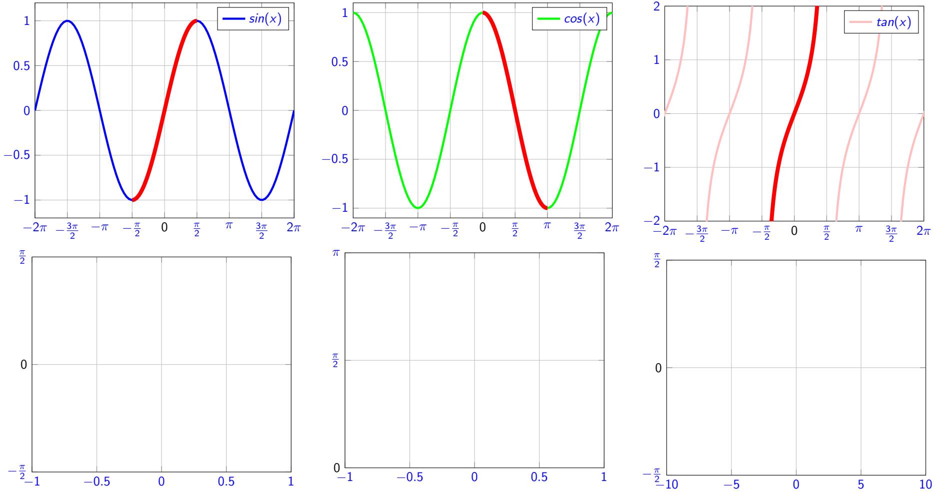
Chapter 3.9: Inverse Trigonometric Functions

Inverse Trigonometric Functions



$$\sin^{-1}(x) = \arcsin(x)$$
$$-1 \leq x \leq 1$$

$$\cos^{-1}(x) = \arccos(x)$$
$$-1 \leq x \leq 1$$

$$\tan^{-1}(x) = \arctan(x)$$
$$x \in \mathbb{R}$$

Derivative of arcsin

$$\frac{d}{dx}\left[\arcsin(x)\right] =$$

1. $\arcsin(-1/2) =$

2. $\cos(\arcsin(x)) =$

Derivative of arctan

$$\frac{d}{dx}\arctan(x) =$$

$$\frac{d}{dx}(\tan(x)) = \frac{d}{dx}\left(\frac{\sin(x)}{\cos(x)}\right) = \frac{1}{\cos(x)^2}$$

$$\cos(x)^2 = \frac{1}{\tan(x)^2 + 1}$$

$$\begin{aligned}\tan(x)^2 + 1 &= \frac{sin(x)^2}{\cos(x)^2} + 1\\ &= \frac{sin(x)^2 + \cos(x)^2}{\cos(x)^2}\\ &= \frac{1}{\cos(x)^2}\end{aligned}$$

Examples

$\boxed{\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}}$ $\boxed{\frac{d}{dx}(\arccos x) = -\frac{1}{\sqrt{1-x^2}}}$ $\boxed{\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}}$

- $\frac{d}{dx}\left[\arccos(x^2)\right] =$
- $\frac{d}{dx}[\ln(\arctan(x))] =$
- $\frac{d}{dx}\left[\arcsin(\sqrt{1-t})\right] =$
- $\frac{d}{dx}[\arctan(\sin(x))] =$
- $\frac{d}{dx}\left[\arctan(\sqrt{x})\right] =$
- $\frac{d}{dx}\left[\ln(1+x^2)\right] =$

More examples for $\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$

Example: Find the tangent line to $y = \arctan(x)$ at $x = 1$.

Example: The poistion of a particle is given by $s(t) = \arctan(t^2)$ where $t \geq 0$. Determine when acceleration is zero.

Hints for in-class set of puzzles #2 on Friday

- Covers material 3.1 – 3.7 (implicit differentiation)
- Five questions
- Not knowing your section gives 2 points reduction.
- No question will involve limits
- Be able to to take derivatives of the following:
  $1, x^n, e^x, \sin x, \cos x, \tan x$
- Know rules for derivatives:
  sum, product, quotient, chain
- Implicit differentiation
- Motion of particle (velocity, acceleration)
- Tangent lines, perpendicular lines
- Given some values of function/derivative, find others
- Understand plots when it comes to derivatives
- Read carefully
- In class - FRIDAY, 12:10pm, Carver 0001. Come ON TIME!
- Still only very basic calculators allowed