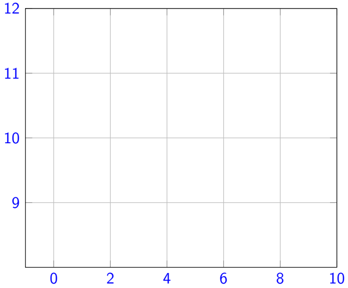
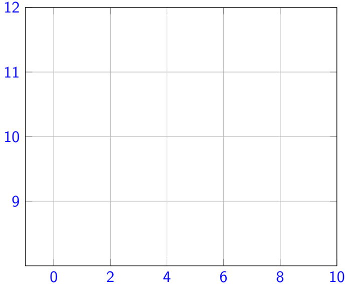
Chapter 4.1: Extreme Values of Functions

Maximum and Minimum

- f has an *absolute maximum* at c if $f(x) \leq f(c)$ for all x.
- f has an *absolute minimum* at c if $f(x) \geq f(c)$ for all x.
- f has an *local maximum* at c if $f(x) \leq f(c)$ for all x *near* c.
- f has an *local minimum* at c if $f(x) \geq f(c)$ for all x *near* c.





Derivatives Help

- If $f'(c) > 0$, then near c our function f is going up.
- If $f'(c) < 0$, then near c our function f is going down.
- *If $f'(c) \neq 0$ and c is NOT ON THE BOUNDARY, then c is not a local min or max.*

A point c is *critical point* if

- $f'(c)$ is undefined
- $f'(c) = 0$
- c is a boundary point

A (local) extreme can only occur at a critical point.

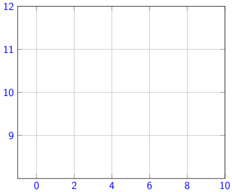
Example: Find all critical points of $x^{2/3}e^{-x/3}$ for $-1 \leq x \leq 5$.

Existence of Extreme Points

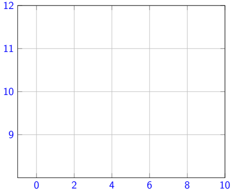
Extreme Value Theorem

A continuous function on a closed and bounded interval (e.g. $a \leq x \leq b$) has an absolute min and absolute max.

Continuous is necessary



Closed and bounded in necessary



If we have a continuous function f on a closed bounded interval, we can find the absolute max and absolute min by the following procedure:

- Find all critical points
- Evaluate f at critical points
- Larges value $=$ absolute max
  smallest value $=$ absolute min

Example: Find absolute min and max of $f(x) = x^2 - x$ on $[0, 1]$

Examples

Find absolute max and min of

$g(x) = 2x^3 - 9x^2 + 12x + 6$ on $[2, 3]$

$f(x) = \sqrt{4 - x^2}$ on $[-2, 1]$

Examples

Find absolute max and min of

$f(x) = |x^2 - 4x - 5|$ for $0 \le x \le 6$

$g(t) = t^8 e^{-t^2}$ for $-1 \le t \le 10$

Examples

Find absolute max and min of $g(x) = 1/(x - 3/4) + \ln(x)$ on $[1, 4]$.