Chapter 4.8: Antiderivatives

Definition

$F(x)$ is an *antiderivative* of $f(x)$ if $F'(x) = f(x)$.

Example: Find anitderivatives:

- $f(x) = 2x$

  $F(x) =$

- $g(x) = \cos(x)$

  $G(x) =$

- $h(x) = 2e^{2x}$

  $H(x) =$

Antiderivatives are unique up to a constant

If $F'(x) = f(x)$ then $(F(x) + C)' = f(x)$, where C is a constant.

If we have some additional information about the antiderivative, we may be able to solve for C and get a unique antiderivative.

Notation

$$\underbrace{\frac{d}{dx}\left(f(x)\right)}_{\text{take derivative}} \qquad \underbrace{\int f(x)\ dx}_{\text{take (all) antiderivatives}}$$

The collection of all antiderivatives is known as *indefinite integral*.

$$\int x^5\ dx$$

$$\int e^{-3x}\ dx$$

$$\int \frac{1}{2\sqrt{x}}\ dx$$

$$\int \frac{1}{x}\ dx$$

$$\int \sin(2x)\ dx$$

Initial value problem

Problem: Find $y(x)$ such that

$$\underbrace{\frac{dy}{dx} = f(x)}_{\text{differential equation}} \quad \text{and} \quad \underbrace{y(x_0) = y_0}_{\text{initial condition}}$$

Solution:

1. Compute *general solution*

$$F(x) + C$$

2. Find *particual solution* by solving

$$y_0 = F(x_0) + C$$

Example:

$$\frac{dy}{dx} = 10 - x, \quad y(0) = -1$$

More entertaining $\int$

- $\int (e^x + 1)^2 \; dx$
- $\int \frac{x^3 - 2x^2 + x - 3}{x^2} \, dx$
- $\int \frac{2x}{1 + x^4} \, dx$
- $\int \tan(x)^2 \, dx$
- $\int \frac{e^{2x} - 1}{e^x + 1} \, dx$