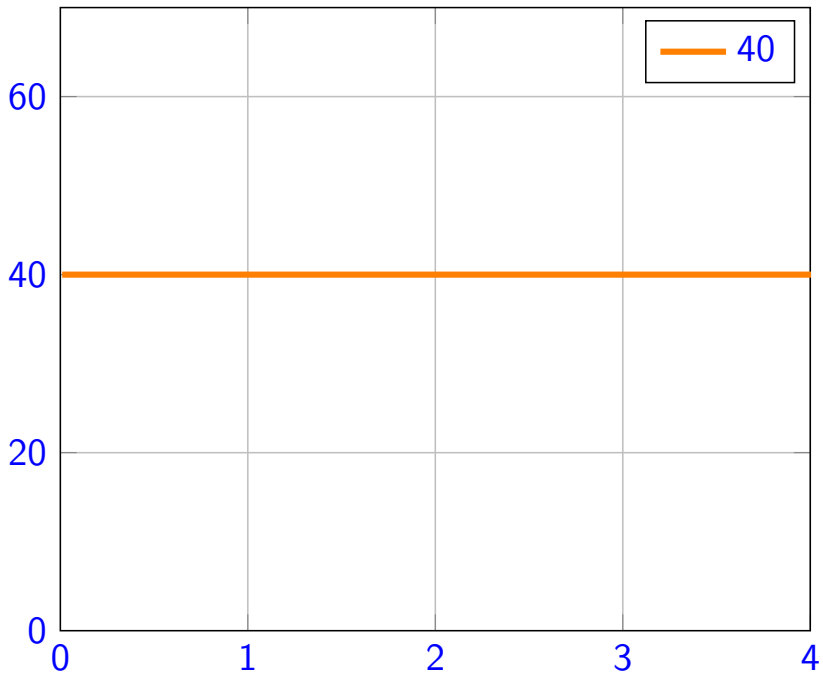
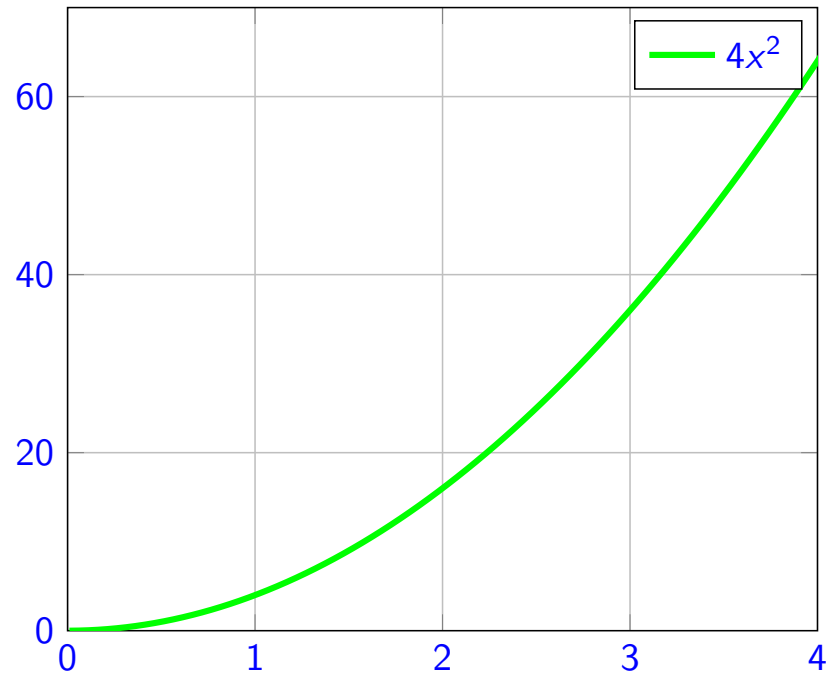
Chapter 5.1: Area and Estimating with Finite Sums

Introduction



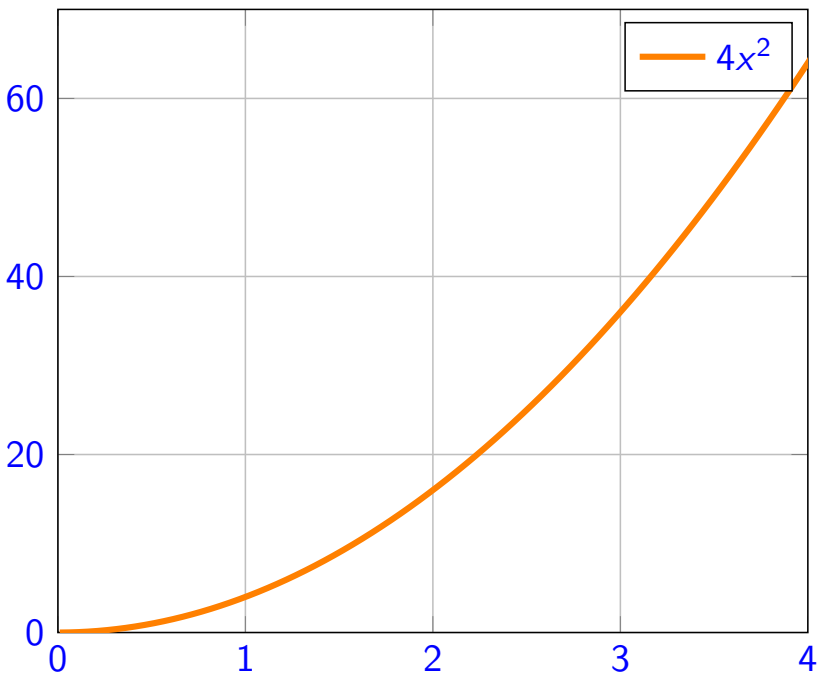


Motivation

Differential Calculus

How are things changing?

Idea: We know how lies are changing.

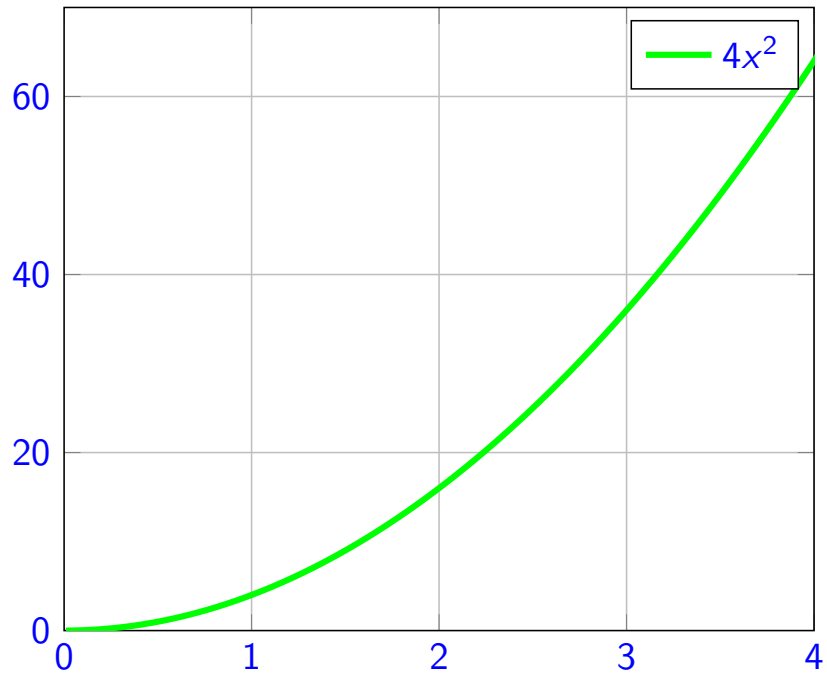


Approximate a function by a line and say it is changing like a line.

Integral Calculus

How much is there?

Problem: Compute the area under $f(x)$.



What is easy for computing area?
How to approximate the area under $f(x)$?

Riemann Sums

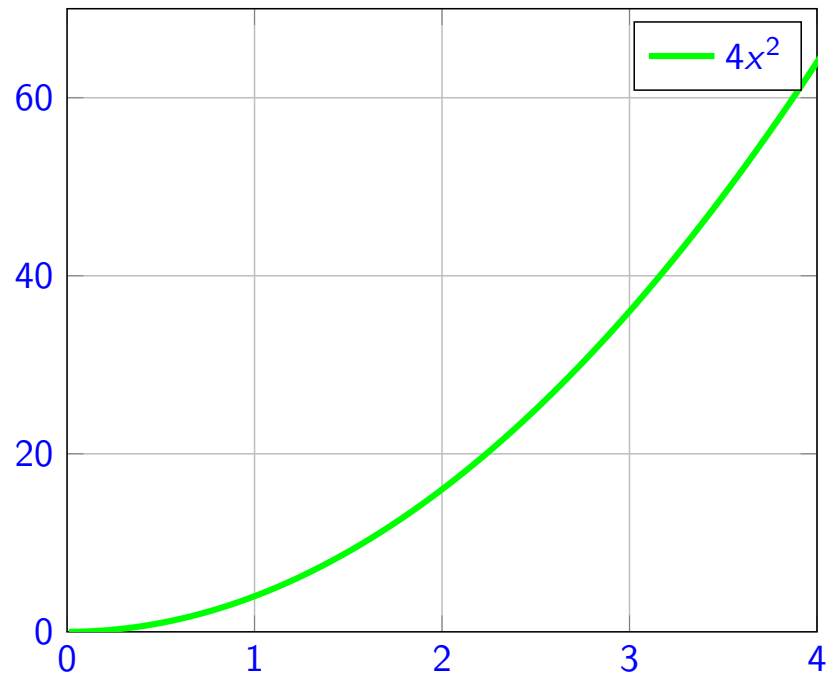
Given a function $y = f(x)$ approximate the area under the curve for $a \leq x \leq b$.

Partition $a \leq x \leq b$ into some (usually equally sized) intervals

$$a = a_0 < a_1 < a_2 < \cdots < a_n = b$$

Approximate the area under $f(x)$ for $a_{i-1} \leq x \leq a_i$ by a rectangle.

width =
height =



Combining the areas of rectangles we get area $\approx$

How to pick x_i?

$$a = a_0 < a_1 < a_2 < \cdots < a_n = b$$

$$a_{i-1} \leq x_i \leq a_i$$

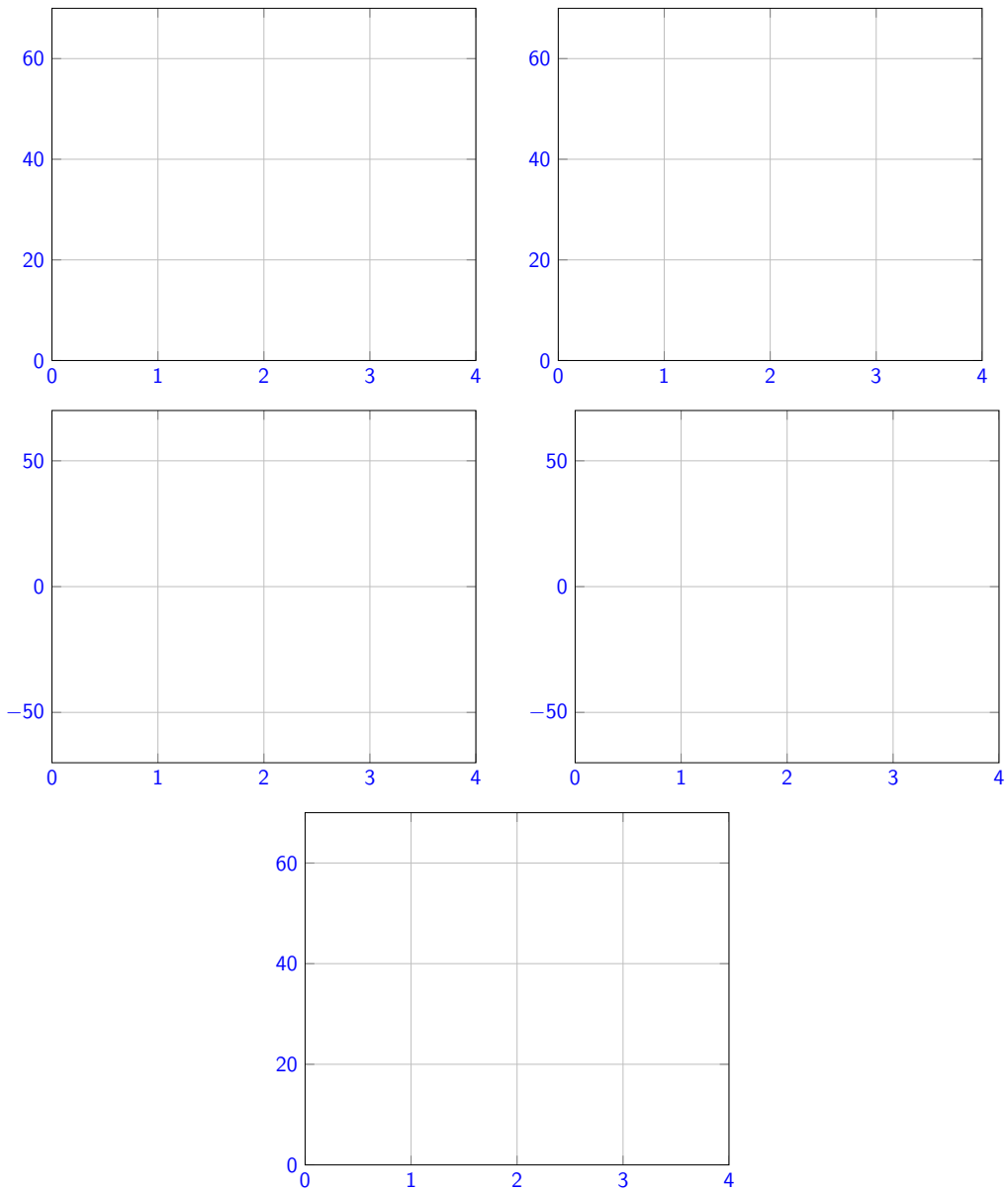
A $\approx f(x_1)\Delta_1 + f(x_2)\Delta_2 + \cdots + f(x_n)\Delta_n$

How to pick x_i?

Options:

- left end: $x_i = a_{i-1}$
- right end: $x_i = a_i$
- midpoint: $x_i = \frac{1}{2}(a_{i-1} + a_i)$
- max value: (varies)
- min value: (varies)

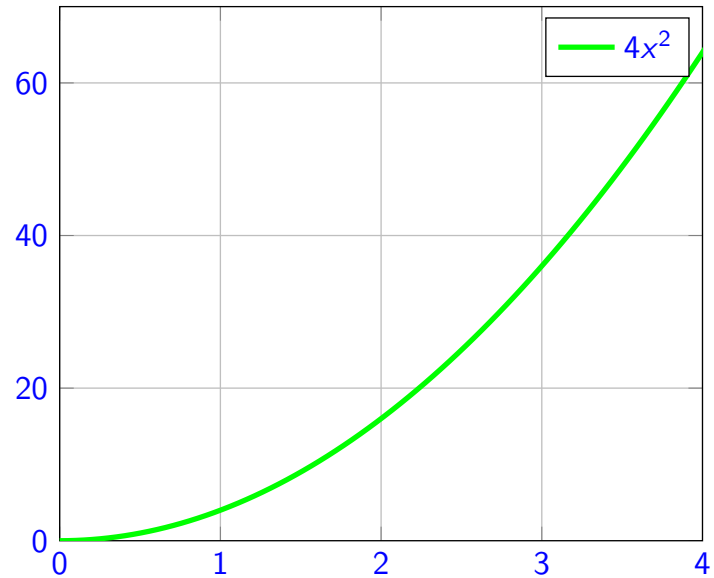
All approximate the area



Example

Estimate the area under $f(x) = 4x^2$ for $0 \leq x \leq 4$ by Reimann sums with four equally spaced intervals and using

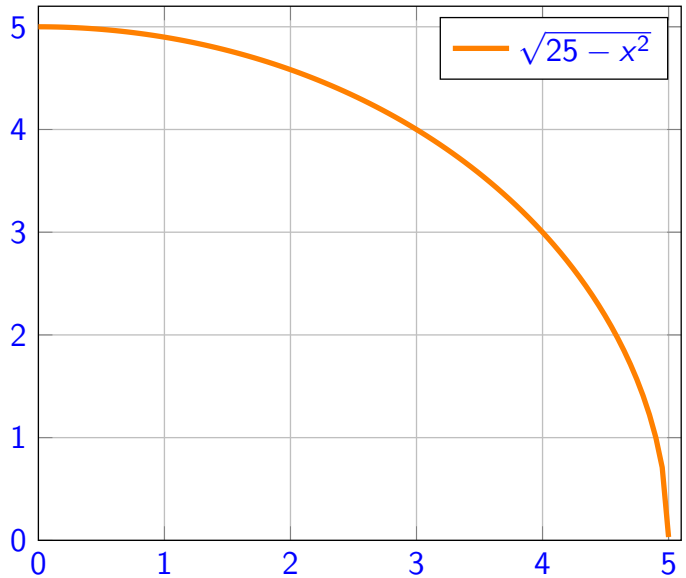
- left end points



- right end points
- midpoints

Example

Estimate the area under the curve $f(x) = \sqrt{25 - x^2}$ for $0 \leq x \leq 5$. Use Riemann sums with five equal intervals and give an upper and lower estimate for the area.



Demo

http://demonstrations.wolfram.com/RiemannSums/