Chapter 5.2: Sigma Notation and Limits of Finite Sums

Sigma Notation

Compressing a big sum into a compact form.

$$\sum_{k=p}^{q} a_k =$$

Index stops at $k = q$

Sigma $\sum_{k=p}^{q} a_k$ Terms in the sum depending on k

Dummy index variable k Index starts at $k = p$

Example:

$$\sum_{k=1}^{4} k = \qquad \sum_{k=1}^{3} 7 =$$

Using $\sum$

Express the following sums in sigma notation:

- $1 + 2 + 3 + 4 + 5 =$

- $1 + 3 + 5 + 7 =$

- $-1 + 2 - 3 + 4 - 5 + 6 =$

- $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} =$

Useful Formulas

- $\sum_{k=p}^{q}(a_k + b_k) =$
- $\sum_{k=p}^{q}(c \cdot a_k) =$
- $\sum_{k=p+r}^{q+r}(a_{k-r}) =$
- $\sum_{k=1}^{n} 1 =$
- $\sum_{k=1}^{n} k =$
- $\sum_{k=1}^{n} k^2 =$
- $\sum_{k=1}^{n} k^3 =$

$$\sum_{k=1}^{n}(2k + 4k^3) =$$

Riemann Sums

Recall approximation of the area under $f(x)$ for $x \in [a, b]$
Pick $a = a_0 < a_1 < \cdots < a_n = b$

$$\text{area } \approx f(x_1)\Delta_1 + f(x_2)\Delta_2 + \cdots + f(x_n)\Delta_n = \sum_{k=1}^{n} f(x_k)\Delta_k$$

Idea: Approximation gets better as $n \to \infty$.

Take n parts of equal size and always take the left point.

$$\text{area } \approx$$

$$\text{Area} =$$

Computing Area - simple

Find the area under $f(x) = 1$ from $x = 0$ to $x = b$ using both geometry and Riemann sums.

Computing Area - maybe?

Find the area under $f(x) = x$ from $x = 0$ to $x = b$ using both geometry and Riemann sums.

Computing Area - finally exciting!

Find the area under $f(x) = 1 - x^2$ from $x = 0$ to $x = 1$ using Riemann sums.