Chapter 5.5: Indefinite Integrals and the Substitution Method

Substitution Method

Helping to integrate chain rule

$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x))g'(x) \qquad \int f'(g(x))g'(x)\ dx = f(g(x)) + C$$

$$\int f'(g(x))g'(x)\ dx = \int f'(u)\ du$$

$$u = g(x)$$

$$du = g'(x)\ dx$$

$$\int 2x\sqrt{x^2+1}\ dx = \int \sqrt{u}\ du$$

$$u = x^2 + 1$$

$$du = 2x\ dx$$

Goal is to simplify the expression.
Look for a function in a function to identify the substitution to make.
Sometimes need to write $dx = h(u)\ du$.

Substitution examples

$$\int 3x^2 e^{x^3}\ dx$$

$$\int \sin(7\theta + 3)\ d\theta$$

$$\int (3x^2 + 1)(x^3 + x)^4\ dx$$

$$\int \frac{z}{\sqrt[3]{z^2 + 1}}\ dz$$

$$\int (x^6 + x)^4\ dx$$

$$\int \sin(x)^3 \cos(x)^3\ dx$$

$$\int \frac{1}{e^x + e^{-x}}\ dx$$

$\int (1 + \sqrt[3]{x})^a\ dx$ for $a \neq -1, -2, -3$.

$$\int 3x^2 e^{x^3}\ dx =$$

$$\int \sin(7\theta + 3)\ d\theta =$$

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$$\int (3x^2 + 1)(x^3 + x)^4\ dx =$$

$$\int \frac{z}{\sqrt[3]{z^2+1}}\, dz =$$

$$\int (x^6 + x)^4\, dx =$$

$$\int \sin(x)^3 \cos(x)^3 \, dx =$$

$$\int \frac{1}{e^x + e^{-x}}\, dx =$$

$$\int (1 + \sqrt[3]{x})^{a} \, dx =$$