Chapter 5.6: Substitution and Area Between Curves

Substitution Method for Definite Integrals by Example

$$\int_0^1 t^3(1+t^4)^3 \, dt =$$

$$\int_0^1 t^3(1+t^4)^3 \, dt =$$

Substitution Method for Definite Integrals by Formula

Dealing with definite integrals

- Compute the antiderivative separately and plug it back with the original variables.
- Update bounds as you go.
  If substitution $u = g(x)$ and $du = g'(x)\,dx$, use

$$\boxed{\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du}$$

Example: $\displaystyle\int_{-2}^{1} (2x+1)e^{(x^2+x)^3}\,dx$

$$\boxed{\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du}$$

$$\int_{-1}^{1} 3x^2\sqrt{x^3+1}\,dx =$$

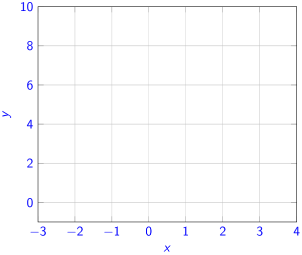
$$\int_0^{\pi/4} \tan(x)\sec^2(x)\,dx =$$

$$\int_0^{\sqrt{3}} \frac{4z}{\sqrt{z^2+1}}\,dz =$$

Area between two curves

If $f(x) \geq g(x)$ on $[a, b]$, then the *area of the region between the curves* on $[a, b]$ is

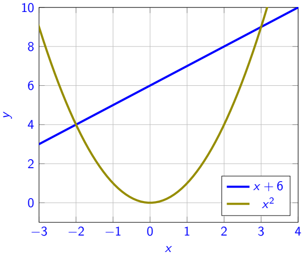
$$\int_a^b [f(x) - g(x)]\, dx.$$



Notice since $f(x) - g(x) \geq 0$, area is positive.

Example; area $= \int_a^b [f(x) - g(x)]\, dx$

Find the area bounded by $y = x + 6$ and $y = x^2$.



More examples

Find the area of the region enclosed by $y = 2 - x^2$ and $y = -x$.

More examples

Find the area of the region bounded by $y = \sqrt{x}$ and $y = x^2$.

Bonus

$$\int_0^{\frac{\pi}{2}} \frac{(\cos x)^{\sin x}}{\cos(x)^{\sin x} + \sin(x)^{\cos x}} \, dx$$

Hint: $\cos(x) = \sin(x + \frac{\pi}{4})$