Chapter 7.2: Exponential Change and Separable Differential Equations

Differential Equations

Sometimes it is easier to figure out how the function is changing than the original function.

Goal: Given information about y' and $y(a) = c$, determine $y(x)$.

Example: Give $y' = \frac{1}{x^2+1}$ and $y(1) = 2$, find $y(x)$.

More Interesting

Case $y' = f(x)$ is easy.

But what about

$$y' = (\text{stuff with } x \text{ and } y)$$

Hard, there are courses focusing on this.
Separable differential equation: isolate x and y

$$\boxed{\frac{dy}{dx} = y' = f(x) \cdot g(y)}$$

Solution plan

- Move y and y' to one side and x to the other side.

$$\frac{1}{g(y)} y' = f(x)$$

- Integrate both sides with respect to x
- Solve y and C using initial condition.

Rabbits like more rabbits. In particular, suppose there are initially 20 rabbits and the rate of change of population is $\frac{1}{10}$ of the current population. Determine the population as a function of time.

Separable equations

$$\frac{dy}{dx} = (1 + y)e^x \text{ and } y(0) = 1$$

$$\boxed{\frac{dy}{dx} = f(x) \cdot g(y)}$$

We find solution using

$$\boxed{\int \frac{1}{g(y)}\, dy = \int f(x)\, dx}$$

Special form $\frac{dy}{dx} = k \cdot y$ and $y(0) = y_0$:

Solve the following separable differential equations.

$\frac{dy}{dx} = e^{x-y}$ and $y(0) = 0$

$\sqrt{2xy} \cdot \frac{dy}{dx} = 1$ and $y(2) = 0$

The half-life of the plutonium-239 is 24360 years. Suppose that we begin with 10g of plutonium-239, find an equation that models that amount of plutonium as a function of years. (Hint: Use exponential growth/decay)