MATH314   HW 2

due **Jan 28** before class, **answer without justification will receive 0 points**. The typing the HW in LaTeX is optional.

If question has (No drawing), you must presents a writeup that is complete and correct without using a picture. If you add a figure to (No drawing) question, it will not be treated negatively but you should not refer to it in the solution.

1: Let S and A be two finite nonempty sets of integers. Define a digraph D with $V(D) = A$, where (x, y) is an arc of D if $x \neq y$ and $y - x \in S$.

(a) Draw the digraph D for $A = \{0, 1, 2, 3, 4\}$ and $S = \{-2, 1, 2, 4\}$.

(b) What can be said about D if A and S consist only of odd integers?

(d) If $|A| = |S| = 5$, how large can the size of D be?

2: Let e be an edge appearing an odd number of times in a closed walk W. Prove that W contains the edges of a cycle through e. (No drawing)

3: Prove that a graph G is bipartite if and only if every subgraph H of G has an independent set consisting of at least half of $V(H)$. (No drawing)

4: Prove that the vertices of any graph can be partitioned into two groups such that for each vertex, at least half of its neighbors ended up in the other group. (No drawing)

5: Let G be a graph of order n. If $\deg(u) + \deg(v) + \deg(w) \geq n - 1$ for every three pairwise nonadjacent vertices u, v and w of G, must G be connected?

6: Show that if G is a connected graph that is not regular, then G contains adjacent vertices u and v such that $\deg(u) \neq \deg(v)$. (No drawing)