MATH314 HW 4

due **Feb 11** before class, **answer without justification will receive 0 points**. The typing the HW in LaTeX is optional.

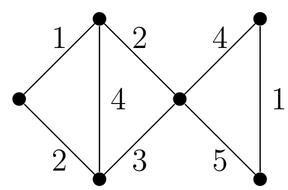
If question has (No drawing), you must presents a writeup that is complete and correct without using a picture. If you add a figure to (No drawing) question, it will not be treated negatively but you should not refer to it in the solution.

1: Prove that Jarník's algorithm is correct. That is, it's output is a minimum spanning tree.

2: A certain tree T of order 35 is known to have 25 vertices of degree 1, two vertices of degree 2, three vertices of degree 4, one vertex of degree 5 and two vertices of degree 6. It also contains two vertices of the same (unknown) degree x. What is x?

3: Show that a tree on n vertices where vertices have degree only 1 and 3 contains $(n-2)/2)$ vertices of degree 3. (No drawing)

4: Apply Kruskal's and Jarník-Prim's algorithm to find minimum spanning tree of



Show how the tree is created after each edge, that means, every algorithm should have a series of pictures how the minimum spanning tree was created.

5: Consider the following algorithm. Let G be a connected graph and $w : E(G) \to \mathbb{R}$. Order all edges of G edges such that $w(e_1) \geq w(e_2) \geq w(e_3) \geq \cdots$. Start with $H = G$. Consider edges one by one according to the ordering, check if e_i is in a cycle in H, and if so remove e_i from H. Is it true that the result of this algorithm is a minimum spanning tree? (No drawing)

6: Every tree is bipartite. Prove that every tree has a leaf in its larger partite set (in both if they have equal size). (No drawing)