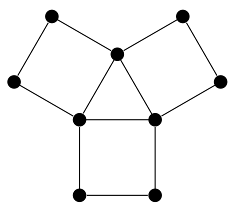
MATH314 HW 5

due **Feb 25** before class, **answer without justification will receive 0 points**. The typing the HW in LaTeX is optional.

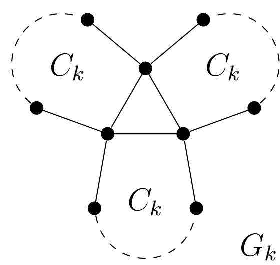
If question has (No drawing), you must presents a writeup that is complete and correct without using a picture. If you add a figure to (No drawing) question, it will not be treated negatively but you should not refer to it in the solution.

1: Show that there is exactly one positive k such that no graph contains exactly k spanning trees.

2: (a) Find the number of spanning trees in the graph G depicted below
(b) Find the number of spanning trees in the graph G_k for $k \geq 5$ depicted below. [Note that (a) is the case where k = 4. C_k stands for a cycle on k vertices.]



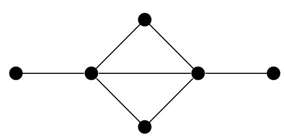
G



G_k

3: Let T and T' be two spanning trees of a connected graph G of order n. Show that there exists a sequence $T = T_0, T_1, \ldots, T_k = T'$ of spanning trees of G such that T_i and T_{i+1} have $n-2$ edges in common for each i with $1 \leq i \leq k-1$.

4: Count the number of spanning trees of the depicted graph using Matrix Tree Theorem.



5: Count the number of spanning trees of K_n using Matrix Tree Theorem.

6: Let G be a graph, which may also contain loops and multiple edges. Let e be an edge that is not a loop. Define $G - e$ as G with the edge e deleted, and $G : e$ as the graphs arising from G by deleting the edge e and subsequent gluing of the end-vertices of e into a single vertex. The other edges are preserved, and so this operation can produce new loops or multiple edges.
Let $T(H)$ denote the number of spanning trees of graph H.
(a) Prove that $T(G) = T(G - e) + T(G : e)$.
(b) Derive the number of spanning trees of the 3-dimensional cube by a calculation based on (a)