

MATH314 HW 6

due **Mar 3** before class, **answer without justification will receive 0 points**. The typing the HW in L^AT_EX is optional.

If question has (No drawing), you must presents a writeup that is complete and correct without using a picture. If you add a figure to (No drawing) question, it will not be treated negatively but you should not refer to it in the solution.

1: Prove or disprove: Let G be a nontrivial graph. For every vertex v of G , $\kappa(G - v) = \kappa(G)$ or $\kappa(G - v) = \kappa(G) - 1$.

2: Show that if G is a connected graph of order at least 3, then its square G^2 is 2-connected.

3: Let uv be an edge of a graph G . **Subdividing** uv means removing the edge and replacing it with path uxv , where x is a new vertex. Show that every 2-connected graph can be obtained from a triangle by adding and subdividing edges.

4: Prove that the symmetric difference of two different edge cuts is an edge cut. (Hint: Draw a picture illustrating the two edge cuts and use it to guide the proof.) Recall that for sets A and B the symmetric difference of A and B is $(A \setminus B) \cup (B \setminus A)$.

5: Let G be a connected k -regular graph with $n = 3k$ vertices. Prove that G is (at least) 2-edge-connected.

6: Let G be a graph. Let T be a graph whose vertices correspond to blocks in G and two vertices in T are adjacent if the corresponding blocks share a vertex. Show that T is a tree.