MATH314 HW 8

due **Mar 31** before class, **answer without justification will receive 0 points**. The typing the HW in $\LaTeX$ is optional.

If question has (No drawing), you must presents a writeup that is complete and correct without using a picture. If you add a figure to (No drawing) question, it will not be treated negatively but you should not refer to it in the solution.

1: Find examples of the following graphs:
(a) graph G_a that is connected, every vertex has degree at least two and G_a is not 2-connected.
(b) graph G_b that is connected, every vertex is in a cycle and G_b is not 2-connected.
(c) graph G_c that is 2-connected, has at least three vertices and it contains three distinct vertices x, y, z such that there is no cycle containing all three vertices x, y and z.

2: There are positions open in seven different divisions of a major company: advertising (a), business (b), computing (c), design (d), experimentation (e), finance (f) and guest relations (g). Six people are applying for some of these positions, namely:

Alvin(A): a,c,f; Bernie (B): a,b,c,d,e,g; Connie (C): c,f;
Donald (D): b,c,d,e,f,g; Edward (E): a,c,f; Frances (F): a,f.

(a) Represent this situation by a bipartite graph.
(b) Is it possible to hire all six applicants for six different positions?

3: A connected bipartite graph G has partite sets U and W, where $|U| = |W| = k \geq 2$. Prove that if every two vertices of U have distinct degrees in G, then G contains a perfect matching.

4: Prove that $\alpha(G) \geq \frac{|V(G)|}{\Delta(G)+1}$ for every graph G.

5: Prove that every maximal matching in a graph G has at least $\alpha'(G)/2$ edges. (Recall that maximum matching has size $\alpha'(G)$. Maximal matching is a set of edges, such that no other edge can be included, but it does not have to be maximum one. For example consider a path on vertices v_1, v_2, v_3, v_4. Edges v_1v_2, v_3v_4 form a maximum matching. Edge v_2v_3 forms a maximal matching since there is no larger matching containing v_2v_3.)

6: Let G be a bipartite graph. Prove that $\alpha(G) = |V(G)|/2$ if and only if G has a perfect matching.