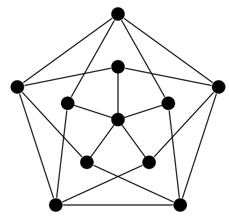
MATH314 **HW 9**

due **Apr 7** before class, **answer without justification will receive 0 points**. The typing the HW in LaTeX is optional.

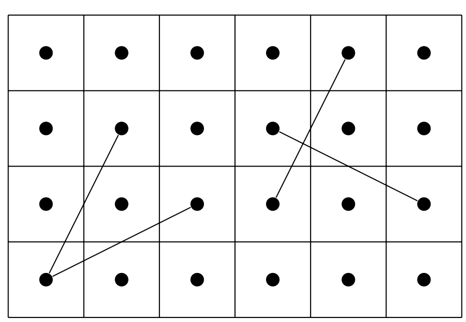
1: Decide if the following graph (called Grötzsch's graph) is Hamiltonian.



2: A mouse eats its way through $3 \times 3 \times 3$ cube of cheese by tunneling through all of the 27 $1 \times 1 \times 1$ subcubes. The mouse starts at one corner of the cube and it always moves on to an uneaten subcube. Can it finish by eating the center of the cube last? (Ignore all gravity related problems - suppose the mouse is on the International Space Station.)

3: Let G be a Hamiltonian bipartite graph, and choose $x, y \in V(G)$. Prove that $G-x-y$ has a perfect matching if and only if x and y are on the opposite side of the bipartition of G. Apply this to prove that deleting two unit squares from an 8 by 8 chessboard leaves a board that can be partitioned into 1 by 2 rectangles if and only if the two missing squares have opposite color.

4: On a chessboard, a **knight** can move from one square to another that differs by 1 in one coordinate and by 2 in the other coordinate, as shown in an example below. Prove that no $4 \times n$ chessboard has a **knight's tour**: a traversal by knight's moves that visits each square once and returns to the start.



Notice that the question can be translated to a graph question and finding a Hamiltonian cycle.

5: Prove that cartesian product of two Hamiltonian graphs in Hamiltonian.

6: Let G be a connected r-regular graph of even order n such that $\overline{G}$ is also connected. Show that

(a) either G or $\overline{G}$ is Eulerian.
(b) either G or $\overline{G}$ is Hamiltonian.