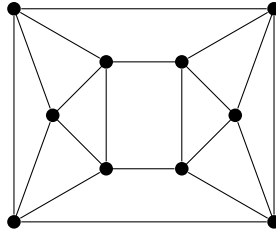


MATH314 HW 10

due **Apr 14** before class, **answer without justification will receive 0 points**. The typing the HW in \LaTeX is optional.

- 1: Find 0-factor, 1-factor, 2-factor, 3-factor and 4-factor in the following graph



- 2: Show that every 3-regular bridgeless graph contains a 2-factor.
- 3: For each $k > 1$ construct a k -regular simple graph having no 1-factor.
- 4: Prove that every graph without isolated vertices has a matching of size at least $\frac{|V(G)|}{1+\Delta(G)}$.
- 5: In an X, Y -bigraph G , the **deficiency** of a set S is $def(S) = |S| - |N(S)|$; note that $def(\emptyset) = 0$. Prove that $\alpha'(G) = |X| - \max_{S \subseteq X} def(S)$. (Hint: Form a bipartite graph G' such that G' has a matching that saturates X if and only if G has a matching of the desired size, and prove that G' satisfies Hall's Condition.)
- 6: Let G be a bridgeless 3-regular graph. Let e be any edge of G . Show that there exists a perfect matching M in G such that $e \notin M$.