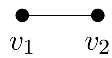
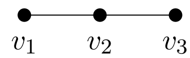
Chapters 1.3 and 1.4 - Classes of Graphs

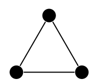
Path P_n of length $n-1$ has vertices $v_1, \ldots, v_n$ and edges $v_i v_{i+1}$ for all $1 \le i \le n-1$.

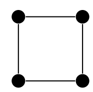
P_1: 

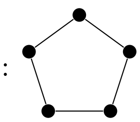
P_2: 

P_3: 

Cycle C_n of length n if obtained from $P_n = v_1, \ldots, v_n$ by adding edge $v_1 v_n$

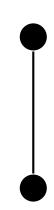
C_3: 

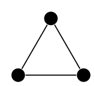
C_4: 

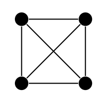
C_5: 

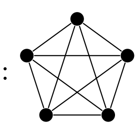
Complete graph K_n has n vertices and for all $u, v \in V(K_n), uv \in E(K_n)$, i.e. *all* edges.

K_1: 

K_2: 

K_3: 

K_4: 

K_5: 

1: What is $|E(K_n)|$?

The **complement** $\overline{G}$ of a graph G is graph where $V(\overline{G}) = V(G)$ and $uv \in E(G)$ iff $uv \notin E(\overline{G})$.

Complement of complete graph is **empty** graph (or **independent set**).

Theorem 1.11 If G is disconnected then $\overline{G}$ is connected.

Graph G is **bipartite** if $V(G) = X \cup Y$, where $G[X]$ and $G[Y]$ are empty graphs.

Theorem 1.12 Graph G is bipartite iff G does not contain an odd cycle.

Complete bipartite graph $K_{m,n}$ is a bipartite graph with parts $|V_1| = m$ and $|V_2| = n$ and for all $u \in V_1$ and $v \in V_2$ we have $uv \in E(K_{m,n})$.

$K_{1,n}$ is called a **star**.

A graph G is k-**partite** if $V(G)$ can be partitioned to $V_1, \dots, V_k$, where $G[V_1]$ induces an empty graph.

A graph is **complete** k-**partite graph** if it is k-partite and maximizes the number of edges.

A **join** $G + H$ is a graph obtained from $G \cup H$ by adding all edges uv, where $u \in V(G)$ and $v \in V(H)$.

A **cartesian product** of G and H, denoted by $G \,\square\, H$ has $V(G \,\square\, H) = V(G) \times V(H) = \{(u, v) : u \in V(G), v \in V(H)\}$ and $E(G \,\square\, H) = \{\{(u, v), (x, y)\} : u = x \text{ and } vy \in E(H) \text{ or } v = y \text{ and } ux \in E(G)\}$.
Note: different notation that in the book!

A **cross product** of G and H, denoted by $G \times H$ has $V(G \times H) = V(G) \times V(H) = \{(u, v) : u \in V(G), v \in V(H)\}$ and $E(G \,\square\, H) = \{\{(u, v), (x, y)\} : ux \in E(G) \text{ and } vy \in E(H)\}$.
Note: different notation that in the book!

2: What is $G \boxtimes H$?

3: *1.21* Draw the graph $3P_4 \cup 2C_4 \cup K_4$.

4: *1.25* Let G be a graph of order 5 or more. Prove that at most one of G and $\overline{G}$ is bipartite.

5: *1.27* For the following pairs G, H of graphs, draw $G + H$, $G \,\square\, H$, $G \times H$.

(a) $G = K_5$ and $H = K_2$;
(b) $G = \overline{K}_5$ and $H = \overline{K}_3$;
(c) $G = C_5$ and $H = K_1$.

6: Find graph on n vertices that maximizes the number of edges but has no K_3 as a subgraph.

Multigraph is a graph where edges can have multiplicities (**multiedges**) and **loops** (edge vv).

Directed graph (or digraph) has edges as ordered pairs rather then sets of size two.

Oriented graph is a graph where edges are oriented (directed).

7: What is the difference between directed graph and oriented graph?

Hypergraph is a graph where edges are any subsets of vertices (not just size 2).

Reading for next time: Chapters 1.3, 1.4