

Chapters 3.1 - Graph Isomorphism; 3.2 - Isomorphism as a Relation

Graph G and H are **isomorphic**, denoted by $G \cong H$ if there exists a bijective mapping $\varphi : V(G) \rightarrow V(H)$ such that

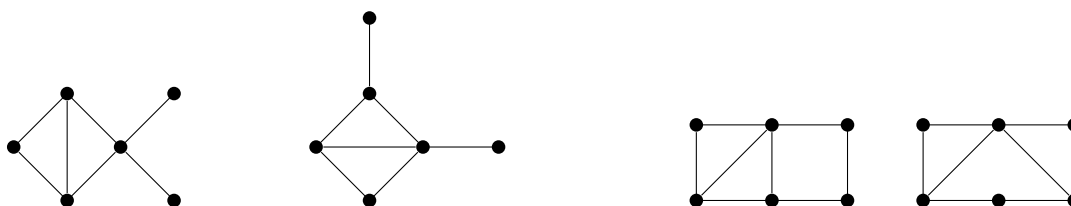
$$uv \in E(G) \text{ if and only if } \varphi(u)\varphi(v) \in E(H) \text{ for all } u, v \in V(G).$$

Graphs G and H are **non-isomorphic** if they are not isomorphic.

1: Find all non-isomorphic graphs on 3 vertices.

A graph G is self-complementary if $G \cong \overline{G}$.

2: Decide if the following pairs are isomorphic.



If two graphs G and H are isomorphic, then

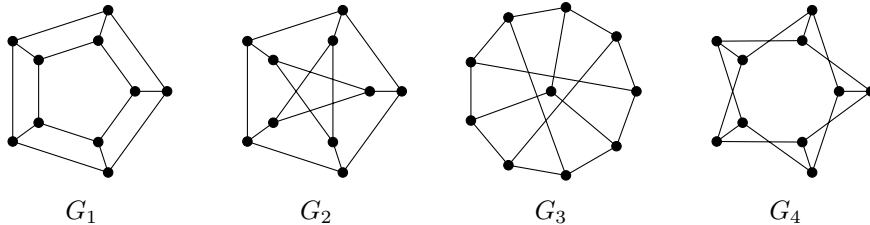
- they have the same order (number of vertices)
- they have the same size (number of edges)
- their complements \overline{G} and \overline{H} are isomorphic
- they have the same number of connected components
- they have the same degree sequences
- G is bipartite iff H is bipartite
- if G' is a(n induced) subgraph of G then there exists H' a(n induced) subgraph of H such that $G' \cong H'$

Theorem 3.6 Isomorphism is an equivalence relation on the set of all graphs.

- 3:** Find all non-isomorphic graphs on 4 vertices.
- 4:** Determine the number of different isomorphisms there are of the graph $K_{3,3}$ to itself.
- 5:** Give example of graphs G and H such that $G \times H$ (the cross product, not Cartesian product) is not bipartite.
- 6:** Draw a graph G which has the following adjacency matrix:

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

- 7:** Find which pairs of the following graphs are isomorphic



- 8:** Give an example of three different (non-isomorphic) graphs of order 5 and size 5.
- 9:** Suppose that there exist two connected graphs G and H and a one-to-one function φ from $V(G)$ onto $V(H)$ such that $d_G(u, v) = d_H(\varphi(u), \varphi(v))$ for every two vertices u and v of G . Prove or disprove: G and H are isomorphic.
- 10:** We are given a collection of n graphs G_1, G_2, \dots, G_n , some pairs of which are isomorphic and some pairs of which are not. Show that there is an even number of these graphs that are isomorphic to an odd number of graphs. [Hint: Construct a graph G with $V(G) = \{v_1, v_2, \dots, v_n\}$, where $v_i v_j \in E(G)$ if and only if G_i is isomorphic to G_j . What are connected components of G ?]

- 11:** *Open problem* Find a graph that is maximizing the number of induced copies of P_4 as subgraphs.

Reading for next time: Chapters 3.3