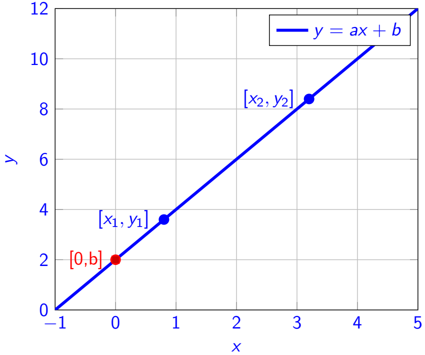
Chapter 2.1. - Rate of Change and Tangents

Line in the plane $y = ax + b$

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

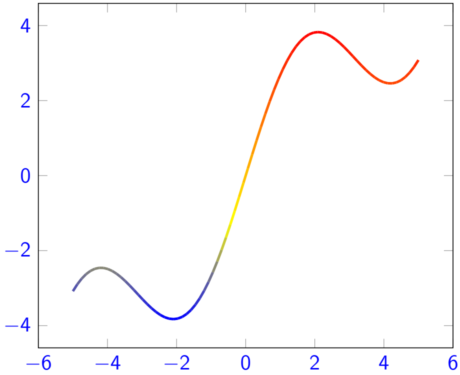


Slope measures *rate of change*.

positive slope = positive rate of change
= going up
negative slope = negative rate of change
= down
zero slope = no change
= going horizontally

Example: Find an equation of the line passing trough points $[2, 1]$ and $[1, 3]$.

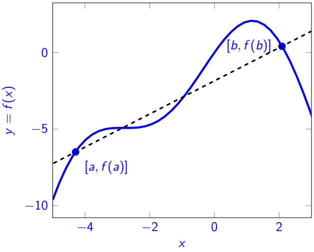
Differential Calculus - Tiny Changes - Earth is Flat



Average Rate of Change

The *average rate of change* for $y = f(x)$ from $x = a$ to $x = b$ is $\frac{f(b)-f(a)}{b-a} = \frac{\Delta y}{\Delta x}$.

Average rate of change is the slope of the *secant line* through $[a, f(a)]$ and $[b, f(b)]$.



Idea: Approximate f from a to b by a line.

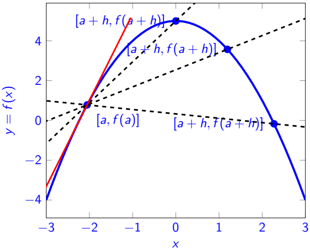
Example: Find the average rate of change for $y = 4x - 19$ from $x = \cos(3)$ to $x = \ln(\pi)$.

Example: Find the average rate of change for $y = x^2 - 2$ from $x = 1$ to $x = 5$.

Instantaneous Rate of Change

The *instantaneous rate of change* for $y = f(x)$ at $x = a$ is the slope of *tangent* to $f(x)$ at $[a, f(a)]$.

Approximate f from a to $a + h$ by a line and try to make h small (zero).



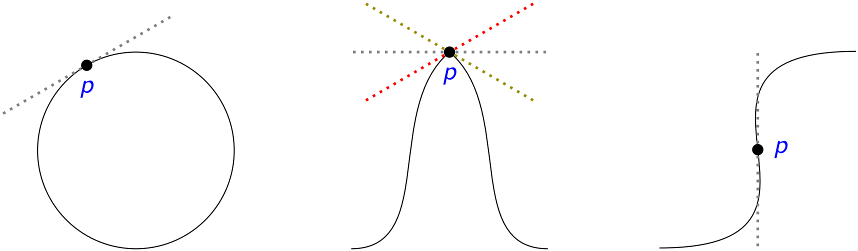
Idea: Approximate $f(x)$ at a by a line.
Secant line goes to tangent line

Example: Find the tangent line for $y = f(x) = 5 - x^2$ at $x = -2$.

$$\begin{aligned}
\frac{\Delta y}{h} &= \frac{f(-2+h) - f(-2)}{(-2+h) - 2} \\
&= \frac{(5 - (-2+h)^2) - (5-4)}{h} \\
&= \frac{(-h^2 + 4h + 1) - 1}{h} \\
&= \frac{-h^2 + 4h}{h} = -h + 4
\end{aligned}$$

Instantaneous rate of change is 4.
Tangent line $y = 4x + b$ must contain point $[-2, 1]$. So it is $y = 4x + 9$.

Tangent Lines Do Not Always Exist



Chapter 2.1 Recap

- Line in the plane is $y = ax + b$
- Average rate of change of $f(x)$ from a to b is the slope of the secant
- Instantaneous rate of change of $f(x)$ is slope of tangent line
- Tangent line can be approximated by secant line
- Computing tangent line using $(x \to a + h)$ and $h \to 0$.
- Tangent line may not be defined