On 3-chosability of triangle free planar graphs

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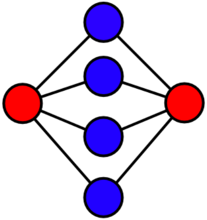
List coloring - quick reminder

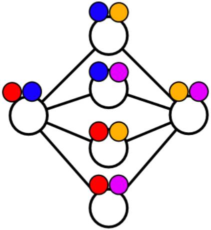
Let G be a graph and C set of colors.

- *coloring* is a mapping $c : V(G) \to C$.
- coloring is *proper* if adjacent vertices have distinct colors
- *chromatic number* $\chi(G)$ is minimum k such that G can be properly colored using k colors.
- *list assignment* is a mapping $L : V(G) \to 2^C$
- *list coloring* (L-coloring) is a coloring c such that $c(v) \in L(v)$ for all $v \in V(G)$
- *choosability* $\chi_\ell(G)$ is minimum k such that if $|L(v)| \geq k$ for all $v \in V(G)$ then G can be properly L-colored

Choosability

- $\chi(G) \leq \chi_\ell(G)$
- $\chi(G) \leq \Delta(G) + 1$ and also $\chi_\ell(G) \leq \Delta(G) + 1$
- Exists graph G: $\chi(G) < \chi_\ell(G)$





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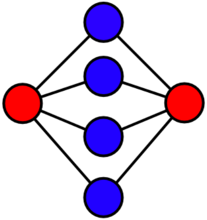

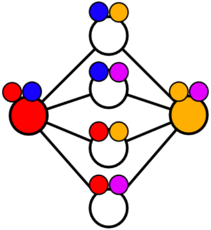

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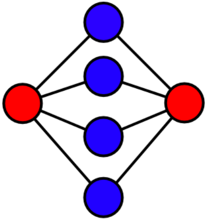
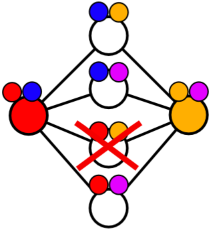




List coloring - motivation to our problem

Theorem (Grötzsch 1959)

Every planar triangle-free graph is 3-colorable.

Theorem (Voigt, 1995)

There exists a planar triangle-free graph which is not 3-choosable

Observation (Kratochvíl and Tuza, 1994)

Every planar triangle-free graph is 4-choosable.

Theorem (Alon and Tarsi, 1992)

Every planar bipartite graph is 3-choosable.

We want to give a sufficient conditions when planar triangle-free graph is 3-choosable.

List coloring - triangle-free graphs

3	4	5	6	7	8	9	authors	year
X		X		X		X	Alon and Tarsi	1992
X	X						Thomassen	1995
X			X	X		X	Zhang and Xu	2004
X		X			X	X	Zhang	2005
X		X	X				Lam, Shiu and Song	2005
X					X	X	Zhang, Xu and Sun	2006
X					X	X	Zhu, Lianying and Wang	2007
X			X	X	X		L.	2009
X				X	X		Dvořák, L. and Škrekovski	2009
X			X	X			Dvořák, L. and Škrekovski	submitted

Google: Mickael Montassier

Our results

Theorem

Every planar triangle-free graph without 4-cycles sharing edges with 4- and 5-cycles is 3-choosable.

Corollary

Every planar graph without 3-,6-, and 7- cycles is 3-choosable.

Theorem

Every planar graph without 3-,7-, and 8- cycles is 3-choosable.

Theorem (Li, 2008)

Every planar triangle-free graph without 4-cycles sharing vertex with 4- and 5-cycles is 3-choosable.

367 - precoloring extension (Thomassen like)

- prove something stronger
- restrict lists of vertices in the outer face
- induction on the number of vertices
- remove some vertices from the outer face and extend the coloring of a smaller graph from induction

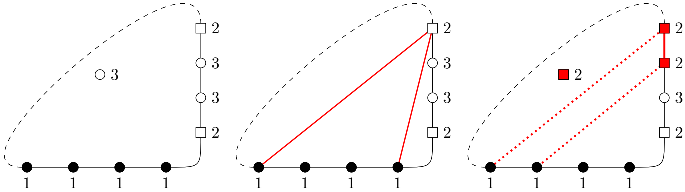
367 - stronger theorem

Theorem

Let G be a triangle-free planar graph without 4-cycles adjacent to 4- and 5-cycles, with outer face C, and P a path of length at most three such that $V(P) \subseteq V(C)$. The graph G can be L-colored for any list assignment L such that

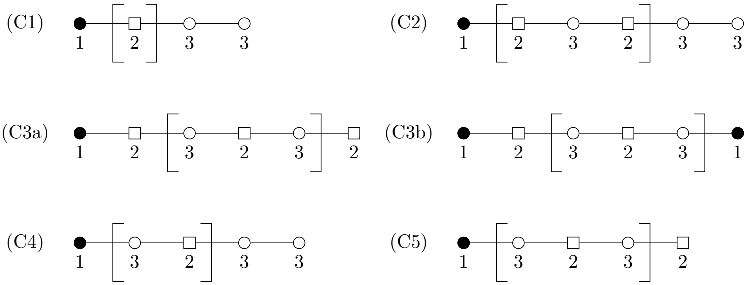
- $|L(v)| = 3$ *for all* $v \in V(G) \setminus V(C)$;
- $2 \leq |L(v)| \leq 3$ *for all* $v \in V(C) \setminus V(P)$;
- $|L(v)| = 1$ *for all* $v \in V(P)$, *and the colors in the lists give a proper coloring of the subgraph of* G *induced by* $V(P)$;
- *the vertices with lists of size two form an independent set; and*
- *each vertex with lists of size two has at most one neighbor in* P.

367 - conditions on pictures

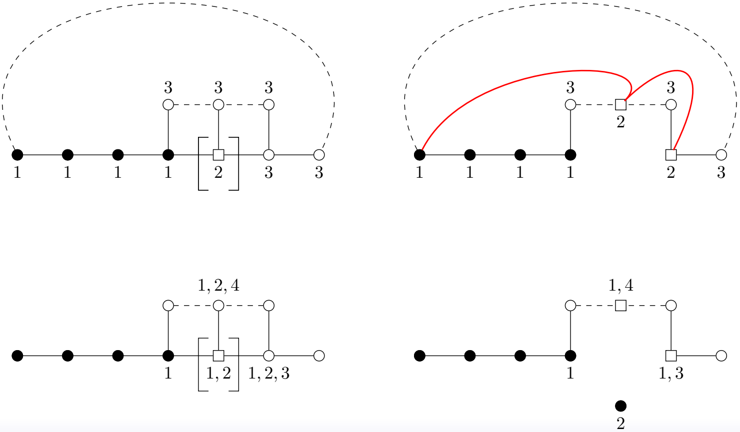


367 - precoloring extension (Thomassen like)

- small cycles (4,5,6,7, 8, 9) induce faces
- dealing with 1-, 2- and 3- chords
- distinguish 5 cases for removing vertices



367 - case (C1)



378 - discharging - basic idea

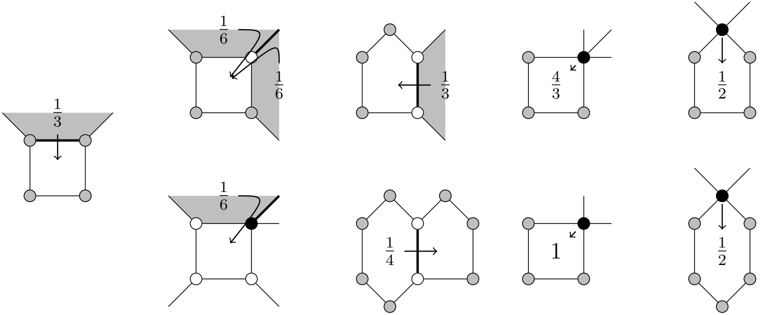
- get the smallest counterexample
- identify reducible configurations
- assign initial charges to faces and vertices (sum < 0 for planar graphs)
- redistribute charges (sum is still the same)
- show that charge of every vertex and face is ≥ 0
- hence sum of charges ≥ 0 and a contradiction

378 - discharging - initial charges

$$ch(v) = 2\deg(v) - 6, ch(f) = l(f) - 6$$

	3	4	5	6	7	8	9
$ch(v)$	0	2	4	6	8	10	12
$ch(f)$		-2	-1	0			3

378 - discharging - discharging rules



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Thank you for your attention