Uniquely K_r-Saturated Graphs

Derrick Stolee

Joint with Stephen G. Hartke

University of Nebraska–Lincoln
s-dstolee1@math.unl.edu
http://www.math.unl.edu/∼s-dstolee1/

January 6, 2012

Supported by NSF grant DMS-0914815 and a University of Nebraska Presidential Fellowship.

H-Saturated Graphs

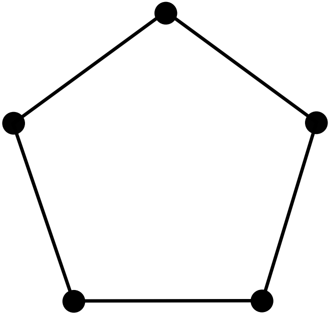
Definition A graph G is ***H*-saturated** if

- G does not contain H as a subgraph. (***H*-free**)
- For every $e \in E(\overline{G})$, $G + e$ contains H as a subgraph.

H-Saturated Graphs

Definition A graph G is ***H*-saturated** if

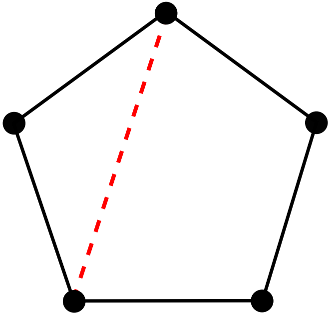
- G does not contain H as a subgraph. (H-**free**)
- For every $e \in E(\overline{G})$, $G + e$ contains H as a subgraph.



H-Saturated Graphs

Definition A graph G is ***H*-saturated** if

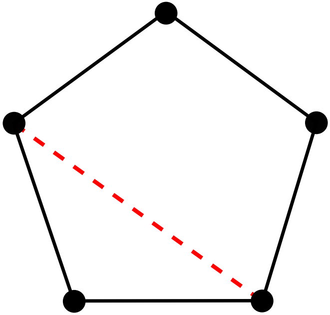
- G does not contain H as a subgraph. (***H*-free**)
- For every $e \in E(\overline{G})$, $G + e$ contains H as a subgraph.



H-Saturated Graphs

Definition A graph G is ***H*-saturated** if

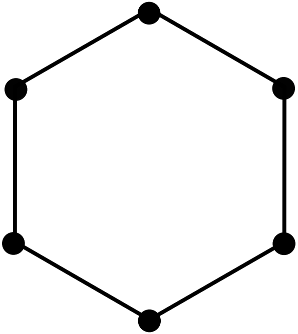
- G does not contain H as a subgraph. (***H*-free**)
- For every $e \in E(\overline{G})$, $G + e$ contains H as a subgraph.



H-Saturated Graphs

Definition A graph G is ***H*-saturated** if

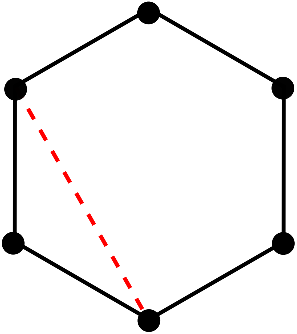
- G does not contain H as a subgraph. (***H*-free**)
- For every $e \in E(\overline{G})$, $G + e$ contains H as a subgraph.



H-Saturated Graphs

Definition A graph G is ***H*-saturated** if

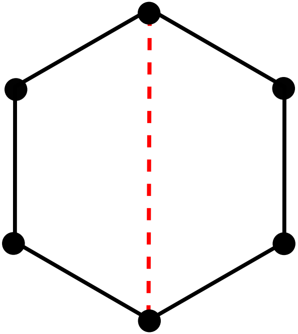
- G does not contain H as a subgraph. (***H*-free**)
- For every $e \in E(\overline{G})$, $G + e$ contains H as a subgraph.



H-Saturated Graphs

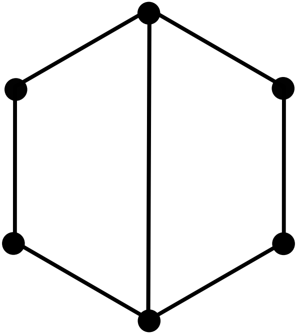
Definition A graph G is ***H*-saturated** if

- G does not contain H as a subgraph. (***H*-free**)
- For every $e \in E(\overline{G})$, $G + e$ contains H as a subgraph.



Definition A graph G is ***H*-saturated** if

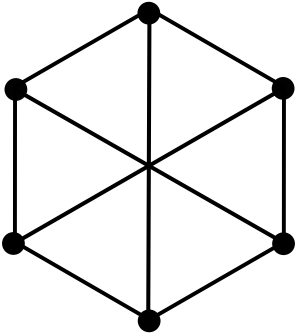
- G does not contain H as a subgraph. (***H*-free**)
- For every $e \in E(\overline{G})$, $G + e$ contains H as a subgraph.



H-Saturated Graphs

Definition A graph G is ***H*-saturated** if

- G does not contain H as a subgraph. (***H*-free**)
- For every $e \in E(\overline{G})$, $G + e$ contains H as a subgraph.

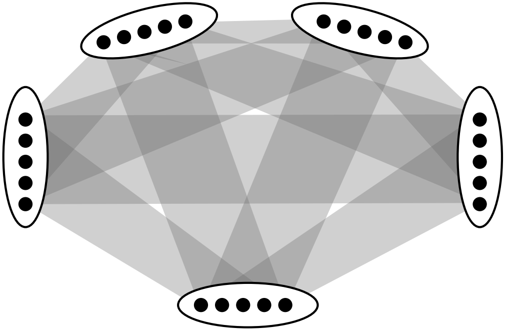


Turán's Theorem

Theorem (Turán, 1941) Let $r \geq 3$. If G is K_r-saturated on n vertices, then G has at most $\left(1 - \frac{1}{r-1}\right)\frac{n^2}{2}$ edges (asymptotically).

Turán's Theorem

Theorem (Turán, 1941) Let $r \geq 3$. If G is K_r-saturated on n vertices, then G has at most $\left(1 - \frac{1}{r-1}\right)\frac{n^2}{2}$ edges (asymptotically).

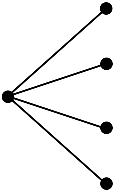


Erdős, Hajnal, and Moon

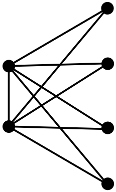
Theorem (Erdős, Hajnal, Moon, 1964) Let $r \geq 2$. If G is K_r-saturated on n vertices, then G has at least $\binom{r-2}{2} + (r-2)(n-r+2)$ edges.

Erdős, Hajnal, and Moon

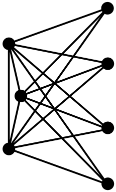
Theorem (Erdős, Hajnal, Moon, 1964) Let $r \geq 2$. If G is K_r-saturated on n vertices, then G has at least $\binom{r-2}{2} + (r-2)(n-r+2)$ edges.



1-book



2-book



3-book

Extremal and Saturation Numbers

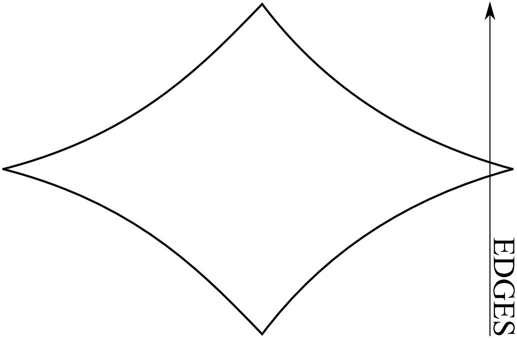
Definition The **extremal number** $\mathrm{ex}(H; n)$ is the **maximum** number of edges in an n-vertex H-saturated graph.

The **saturation number** $\mathrm{sat}(H; n)$ is the **minimum** number of edges in an n-vertex H-saturated graph.

Extremal and Saturation Numbers

Definition The **extremal number** $\mathrm{ex}(H; n)$ is the **maximum** number of edges in an n-vertex H-saturated graph.

The **saturation number** $\mathrm{sat}(H; n)$ is the **minimum** number of edges in an n-vertex H-saturated graph.

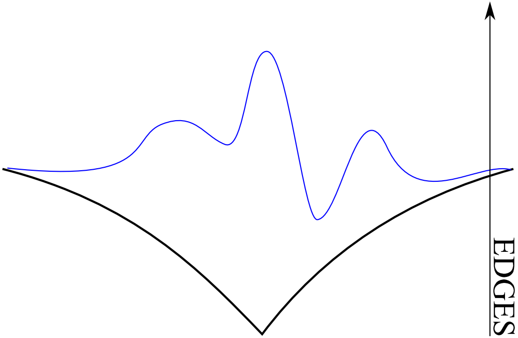


All graphs

Extremal and Saturation Numbers

Definition The **extremal number** $\mathrm{ex}(H;n)$ is the **maximum** number of edges in an n-vertex H-saturated graph.

The **saturation number** $\mathrm{sat}(H;n)$ is the **minimum** number of edges in an n-vertex H-saturated graph.

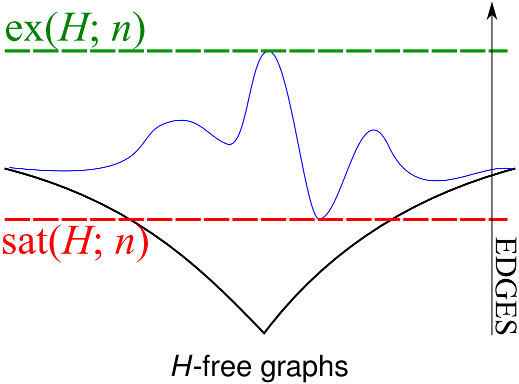


H-free graphs

Extremal and Saturation Numbers

Definition The **extremal number** $ex(H; n)$ is the **maximum** number of edges in an n-vertex H-saturated graph.

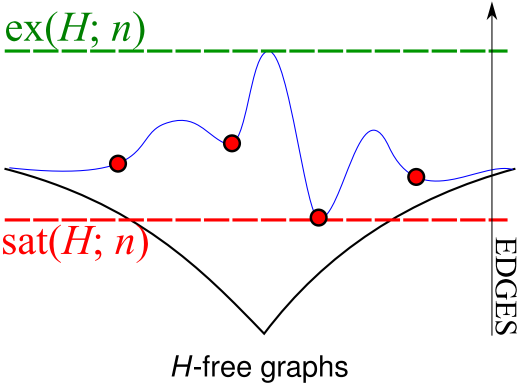
The **saturation number** $sat(H; n)$ is the **minimum** number of edges in an n-vertex H-saturated graph.



Extremal and Saturation Numbers

Definition The **extremal number** $\mathrm{ex}(H; n)$ is the **maximum** number of edges in an n-vertex H-saturated graph.

The **saturation number** $\mathrm{sat}(H; n)$ is the **minimum** number of edges in an n-vertex H-saturated graph.



Uniquely H-Saturated Graphs

The Turán graph has **many** copies of K_r when an edge is added.

The books have **exactly one** copy of K_r when an edge is added.

Uniquely H-Saturated Graphs

The Turán graph has **many** copies of K_r when an edge is added.

The books have **exactly one** copy of K_r when an edge is added.

Definition A graph G is **uniquely H-saturated** if G does not contain H as a subgraph and for every edge $e \in \overline{G}$ admits **exactly one** copy of H in $G + e$.

Uniquely C_k-Saturated Graphs

Lemma (Cooper, Lenz, LeSaulnier, Wenger, West, 2011)

The uniquely C_3-saturated graphs are either **stars** or **Moore graphs** of diameter 2 and girth 5.

Uniquely C_k-Saturated Graphs

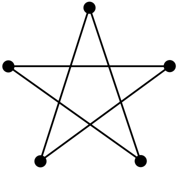
Lemma (Cooper, Lenz, LeSaulnier, Wenger, West, 2011)
The uniquely C_3-saturated graphs are either **stars** or **Moore graphs** of diameter 2 and girth 5.

Theorem (Hoffman, Singleton, 1964) There are a **finite number** of Moore graphs of diameter 2 and girth 5.

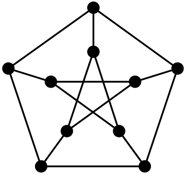
Uniquely C_k-Saturated Graphs

Lemma (Cooper, Lenz, LeSaulnier, Wenger, West, 2011)
The uniquely C_3-saturated graphs are either **stars** or **Moore graphs** of diameter 2 and girth 5.

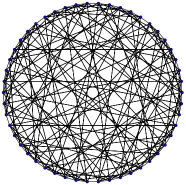
Theorem (Hoffman, Singleton, 1964) There are a **finite number** of Moore graphs of diameter 2 and girth 5.



C_5



Petersen



Hoffman–Singleton

?

57-Regular
Order 3250

Uniquely C_k-Saturated Graphs

Theorem (Cooper, Lenz, LeSaulnier, Wenger, West, 2011)
There are a finite number of uniquely C_4-saturated graphs.

Uniquely C_k-Saturated Graphs

Theorem (Cooper, Lenz, LeSaulnier, Wenger, West, 2011)
There are a finite number of uniquely C_4-saturated graphs.

Theorem (Wenger, 2010)
The only uniquely C_5-saturated graphs are **friendship graphs**.

Uniquely C_k-Saturated Graphs

Theorem (Cooper, Lenz, LeSaulnier, Wenger, West, 2011)
There are a finite number of uniquely C_4-saturated graphs.

Theorem (Wenger, 2010)
The only uniquely C_5-saturated graphs are **friendship graphs**.

Theorem (Wenger, 2010)
For $k \in \{6, 7, 8\}$, no uniquely C_k-saturated graph exists.

Conjecture (Wenger, 2010)
For $k \geq 9$, no uniquely C_k-saturated graph exists.

Uniquely K_r-Saturated Graphs

We consider the case where $H = K_r$ (an ***r*-clique**) for $r \geq 4$.

We consider the case where $H = K_r$ (an ***r*-clique**) for $r \geq 4$.

($K_3 \cong C_3$)

Uniquely K_r-Saturated Graphs

We consider the case where $H = K_r$ (an ***r*-clique**) for $r \geq 4$.

($K_3 \cong C_3$)

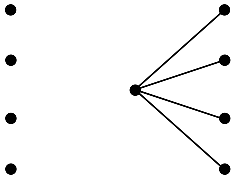
Definition A graph G is **uniquely K_r-saturated** if G does not contain an ***r*-clique** and for every edge $e \in \overline{G}$ there is *exactly one* r-clique in $G + e$.

Dominating Vertices

Adding a dominating vertex to a uniquely K_r-saturated graph creates a uniquely K_{r+1}-saturated graph.

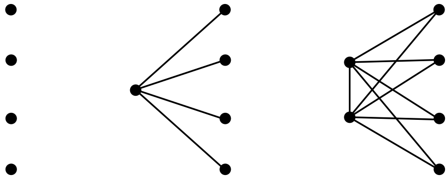
Dominating Vertices

Adding a dominating vertex to a uniquely K_r-saturated graph creates a uniquely K_{r+1}-saturated graph.



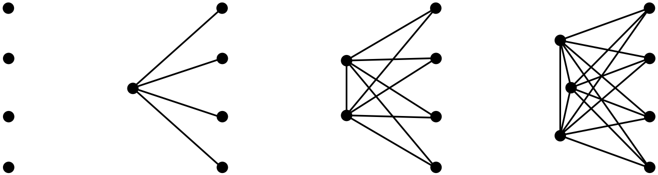
Dominating Vertices

Adding a dominating vertex to a uniquely K_r-saturated graph creates a uniquely K_{r+1}-saturated graph.



Dominating Vertices

Adding a dominating vertex to a uniquely K_r-saturated graph creates a uniquely K_{r+1}-saturated graph.



Dominating Vertices

Call uniquely K_r-saturated graphs without a dominating vertex

***r*-primitive**.

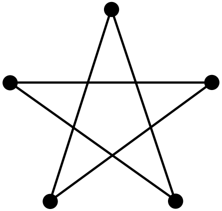
2**-primitive** graphs are **empty graphs**.

Dominating Vertices

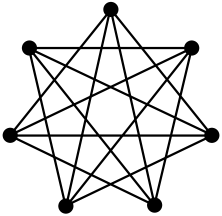
Call uniquely K_r-saturated graphs without a dominating vertex

***r*-primitive**.

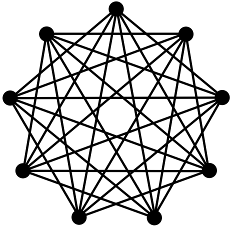
For $r \geq 1$, $\overline{C_{2r-1}}$ is r-primitive.



$\overline{C_5}$



$\overline{C_7}$

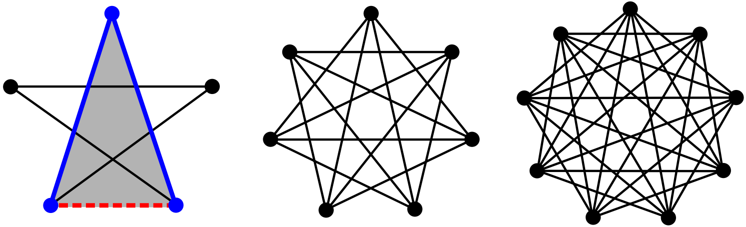


$\overline{C_9}$

Call uniquely K_r-saturated graphs without a dominating vertex

r-primitive.

For $r \geq 1$, $\overline{C_{2r-1}}$ is r-primitive.

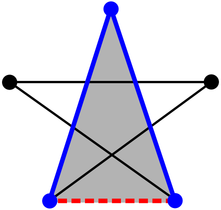


$\overline{C_5}$ $\overline{C_7}$ $\overline{C_9}$

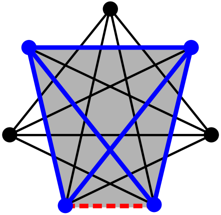
Call uniquely K_r-saturated graphs without a dominating vertex

r-primitive.

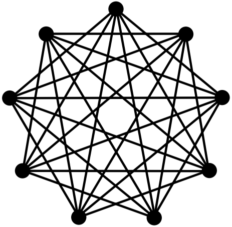
For $r \geq 1$, $\overline{C_{2r-1}}$ is r-primitive.



$\overline{C_5}$



$\overline{C_7}$



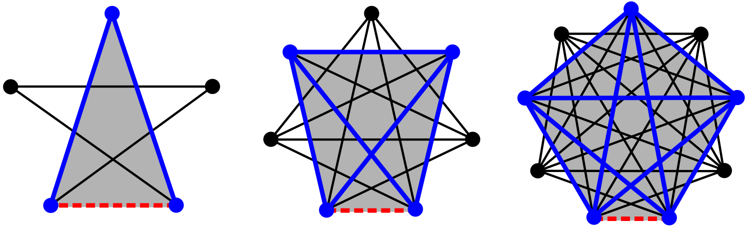
$\overline{C_9}$

Dominating Vertices

Call uniquely K_r-saturated graphs without a dominating vertex

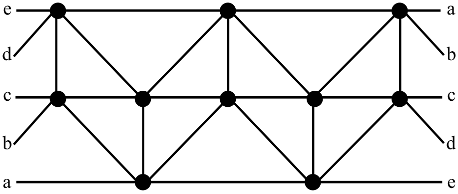
r-**primitive**.

For $r \geq 1$, $\overline{C_{2r-1}}$ is r-primitive.

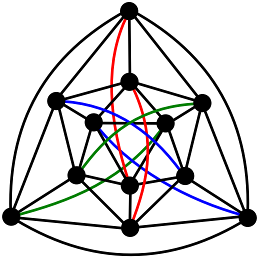


$\overline{C_5}$ $\overline{C_7}$ $\overline{C_9}$

Uniquely K_4-Saturated Graphs



10 vertices



12 vertices

Previously known 4-primitive graphs (Collins, Cooper, Kay, 2010)

Two Questions

Two Questions

1. Fix $r \geq 3$. Are there a **finite number** of r-primitive graphs?

Two Questions

1. Fix $r \geq 3$. Are there a **finite number** of r-primitive graphs?

2. Is every r-primitive graph **regular**?

Variables

We search for uniquely K_r-saturated graphs on vertices $\{v_1, \dots, v_n\}$.

Variables

We search for uniquely K_r-saturated graphs on vertices $\{v_1, \ldots, v_n\}$.

Use variables $x_{i,j} \in \{0, 1, *\}$ where

- $x_{i,j} = 0$ iff $v_i v_j \notin E(G)$.
- $x_{i,j} = 1$ iff $v_i v_j \in E(G)$.
- $x_{i,j} = *$ is **unassigned**.

Symmetries of the System

The constraints

- There is no r-clique in G.
- Every non-edge e of G has exactly one r-clique in $G + e$.

are independent of vertex labels.

Symmetries of the System

The constraints

- There is no r-clique in G.
- Every non-edge e of G has exactly one r-clique in $G+e$.

are independent of vertex labels.

The permutations in S_n permute the variables $x_{i,j}$ by permuting the indices:

$$\sigma \in S_n, \qquad x_{i,j} \xrightarrow{\sigma} x_{\sigma(i),\sigma(j)}.$$

Orbital Branching

Orbital branching reduces the number of isomorphic duplicates.

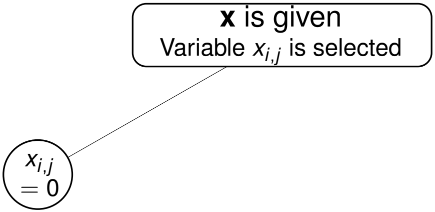
Generalizes **branch-and-bound** strategy.

Introduced by Ostrowski, Linderoth, Rossi, and Smriglio (2007) for **symmetric** optimization problems such as covering and packing.

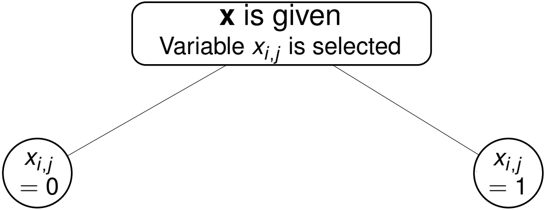
Branch-and-Bound

x is given
Variable $x_{i,j}$ is selected

Branch-and-Bound



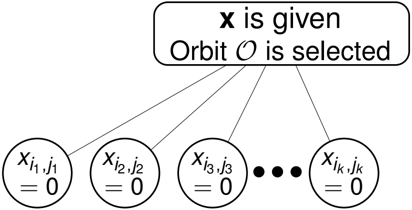
Branch-and-Bound



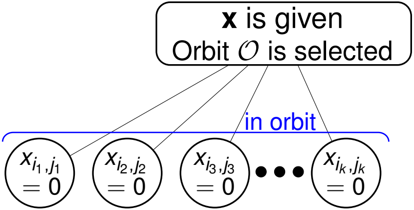
Orbital Branching

$\mathbf{x}$ is given
Orbit $\mathcal{O}$ is selected

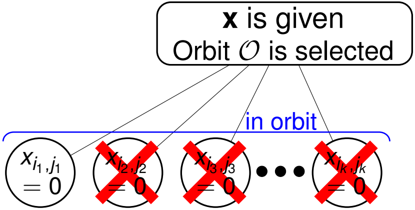
Orbital Branching



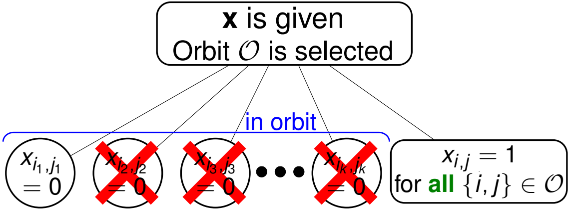
Orbital Branching



Orbital Branching



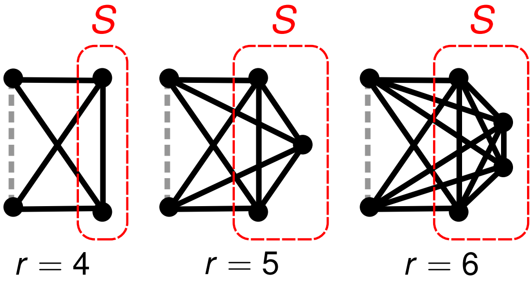
Orbital Branching



K_r-Completions

For every non-edge we add, we add a K_r-**completion**:

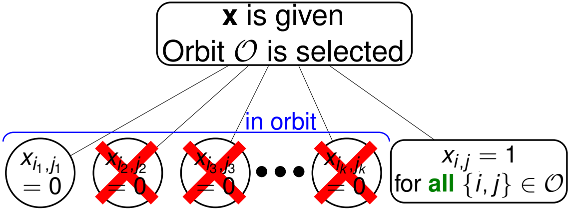
$x_{i,j} = 0$ **if and only if** there exists a set $S \subset [n]$, $|S| = r - 2$, so that $x_{i,a} = x_{j,a} = x_{a,b} = 1$ for all $a, b \in S$.



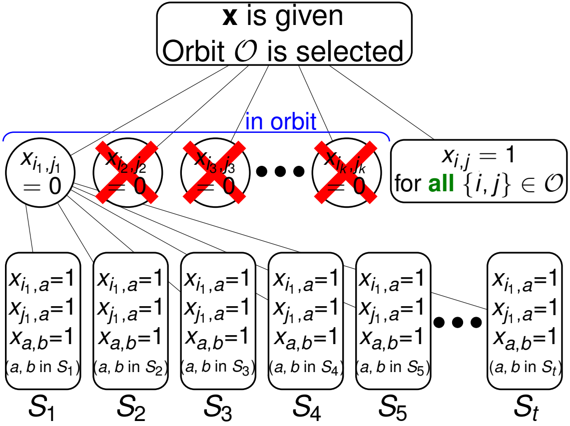
Orbital Branching with K_r-Completions

$\mathbf{x}$ is given
Orbit $\mathcal{O}$ is selected

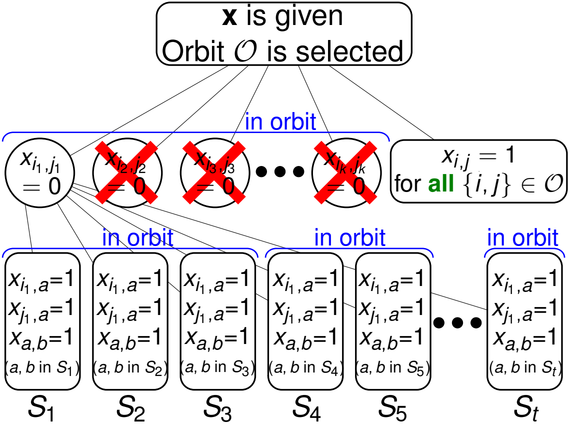
Orbital Branching with K_r-Completions



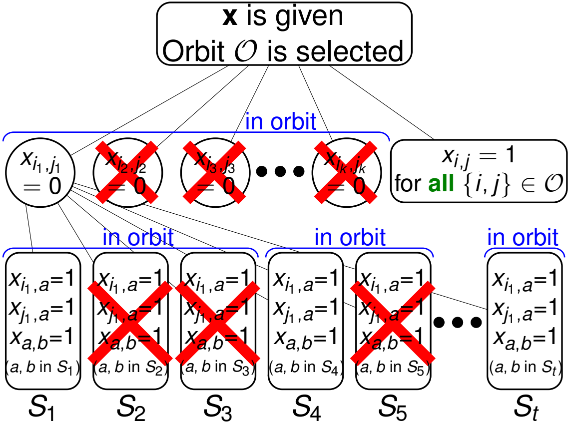
Orbital Branching with K_r-Completions

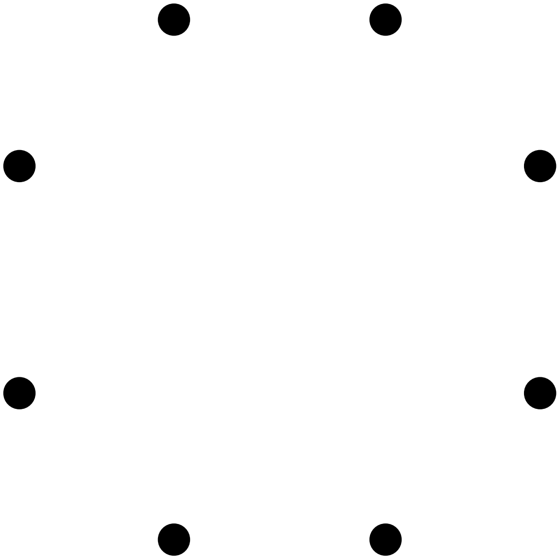


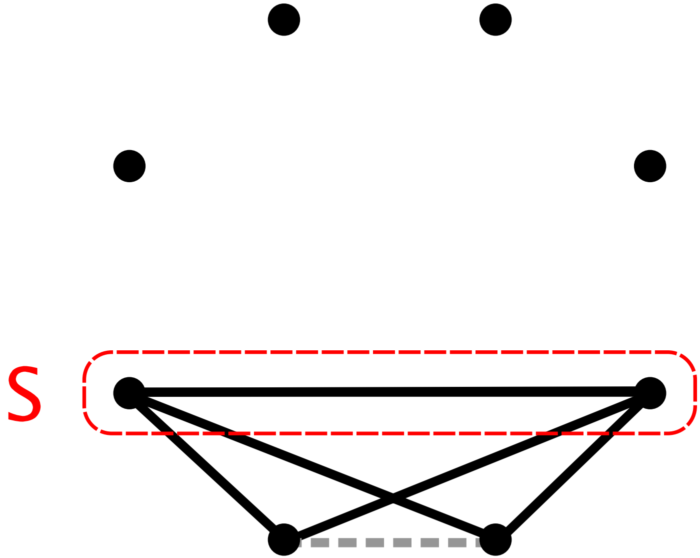
Orbital Branching with K_r-Completions

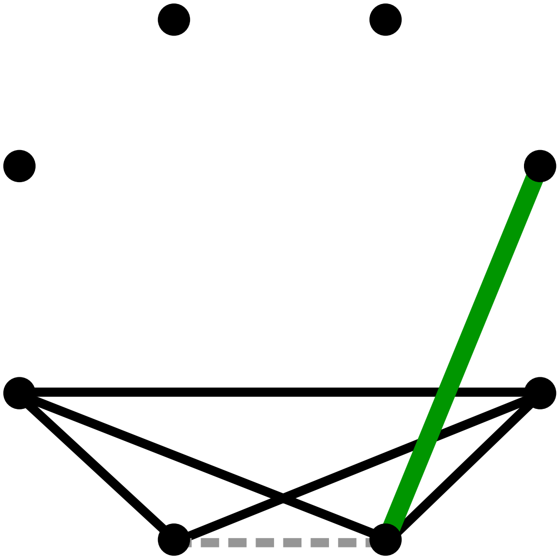


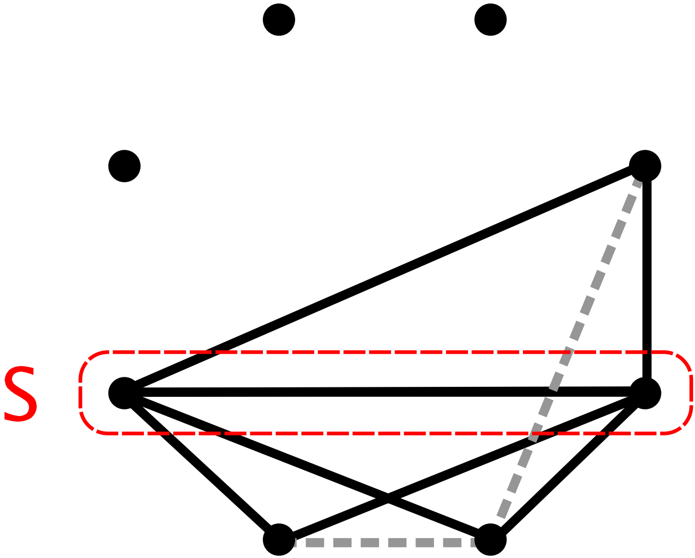
Orbital Branching with K_r-Completions

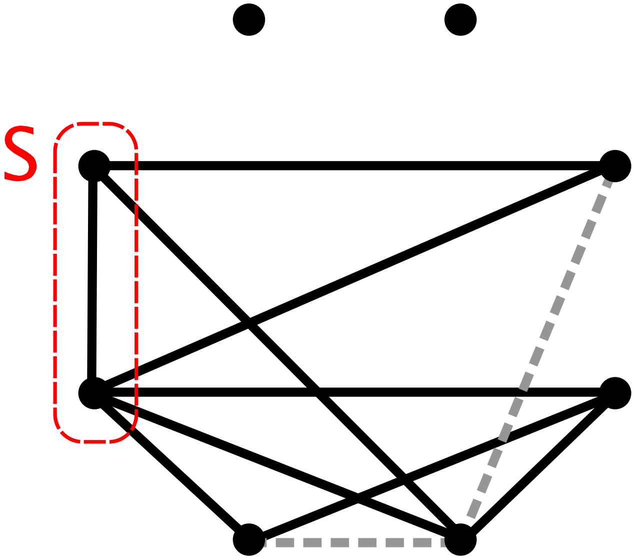


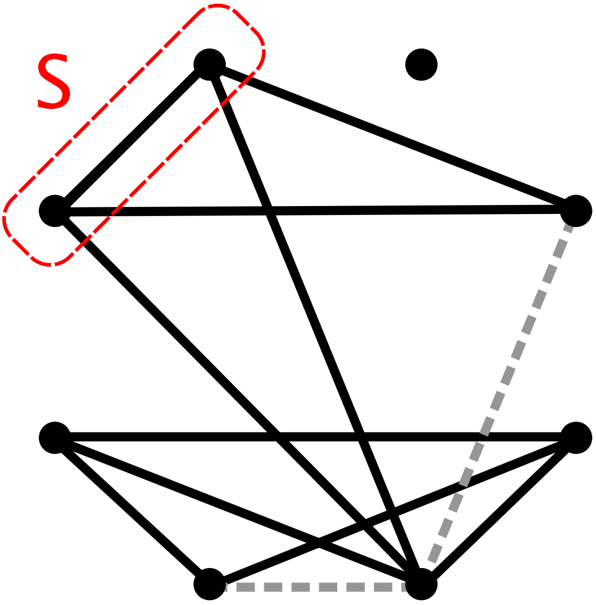


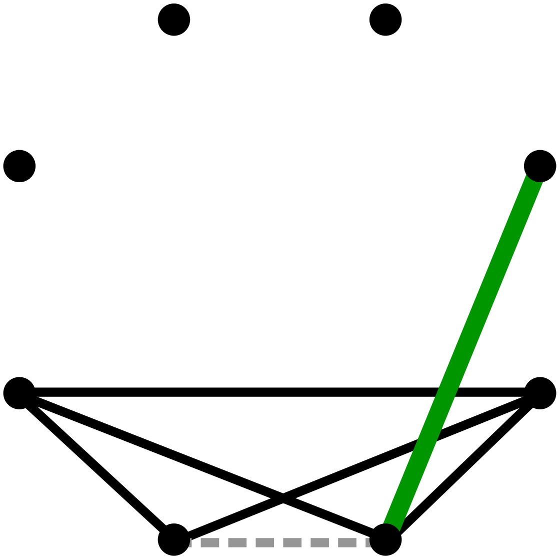

S

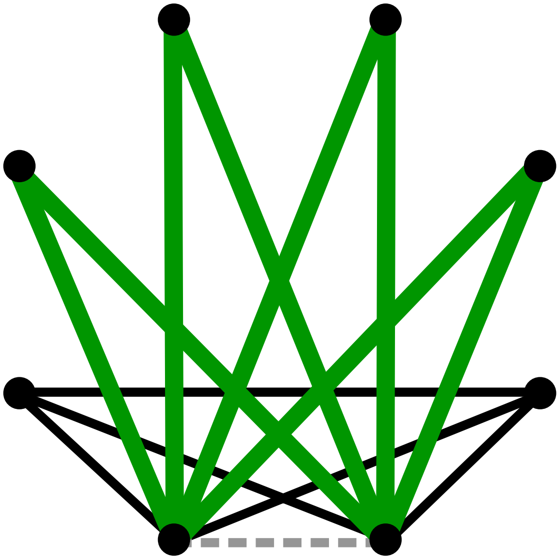


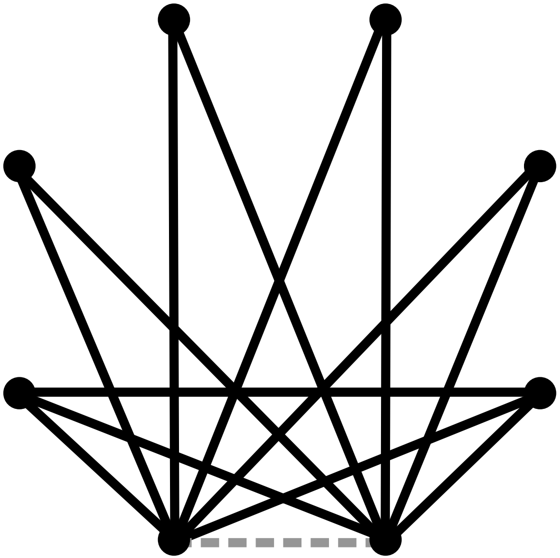

S


S


S







Exhaustive Search Times

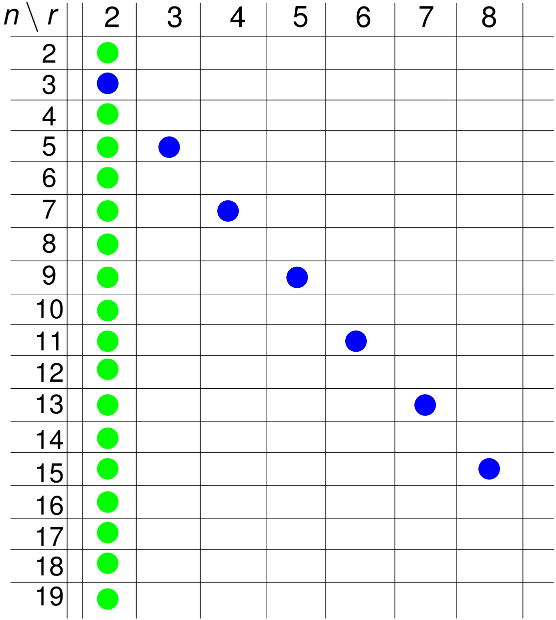
n	$r = 4$	$r = 5$	$r = 6$	$r = 7$	$r = 8$
10	0.10 s	0.37 s	0.13 s	0.01 s	0.01 s
11	0.68 s	5.25 s	1.91 s	0.28 s	0.09 s
12	4.58 s	1.60 m	25.39 s	1.97 s	1.12 s
13	34.66 s	34.54 m	6.53 m	59.94 s	20.03 s
14	4.93 m	10.39 h	5.13 h	20.66 m	2.71 m
15	40.59 m	23.49 d	10.08 d	12.28 h	1.22 h
16	6.34 h	1.58 y	1.74 y	34.53 d	1.88 d
17	3.44 d			8.76 y	115.69 d
18	53.01 d				
19	2.01 y				
20	45.11 y				

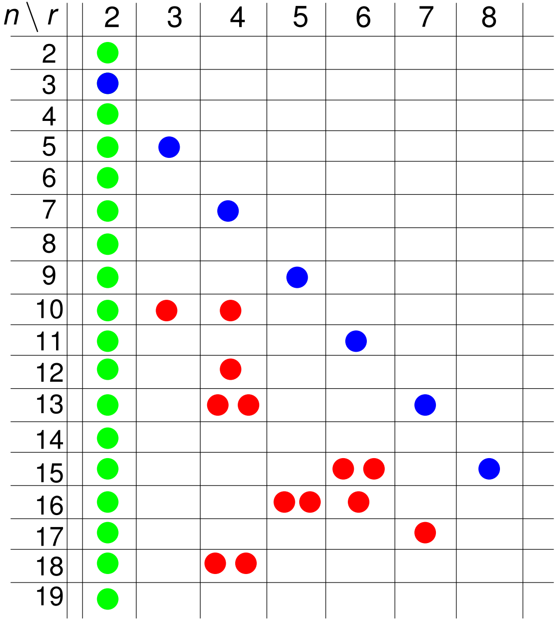
Total CPU times using Open Science Grid.

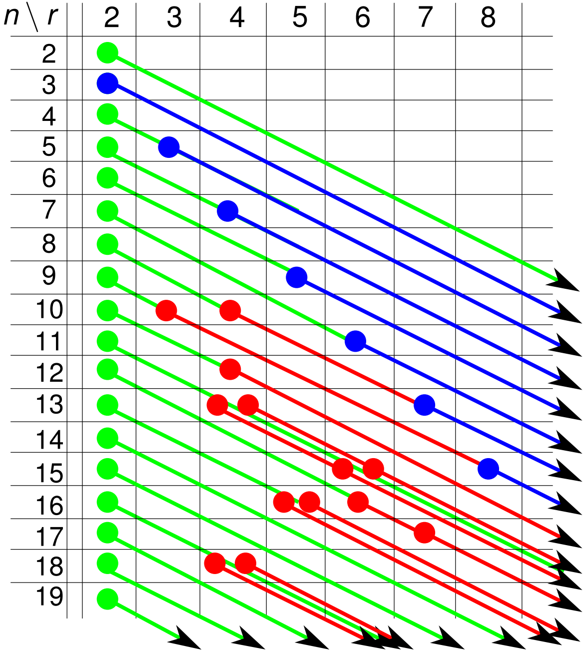
(Recall: $\approx 8.83 \times 10^{18}$ connected graphs of order 20)

$n \setminus r$		2	3	4	5	6	7	8	
2									
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									
13									
14									
15									
16									
17									
18									
19									

$n \setminus r$	2	3	4	5	6	7	8
2	●						
3	●						
4	●						
5	●						
6	●						
7	●						
8	●						
9	●						
10	●						
11	●						
12	●						
13	●						
14	●						
15	●						
16	●						
17	●						
18	●						
19	●						


$n \setminus r$
2 3 4 5 6 7 8
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19

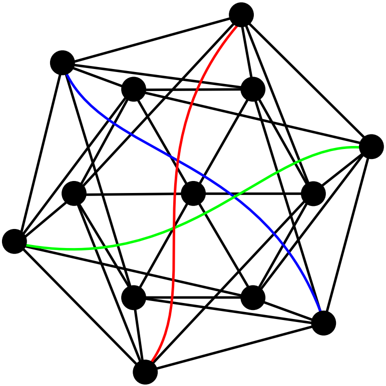

n \ r
2
3
4
5
6
7
8
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19



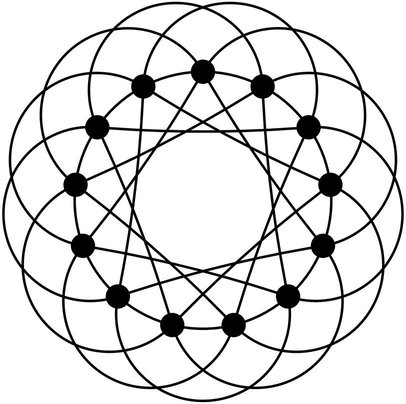
n \ r
2
3
4
5
6
7
8
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19

4-Primitive Graphs

$n = 13$



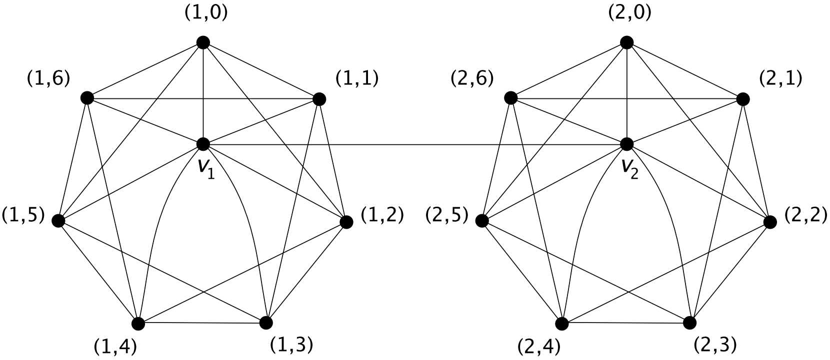
$G_{13}^{(A)}$



$\text{Payley}(13)$

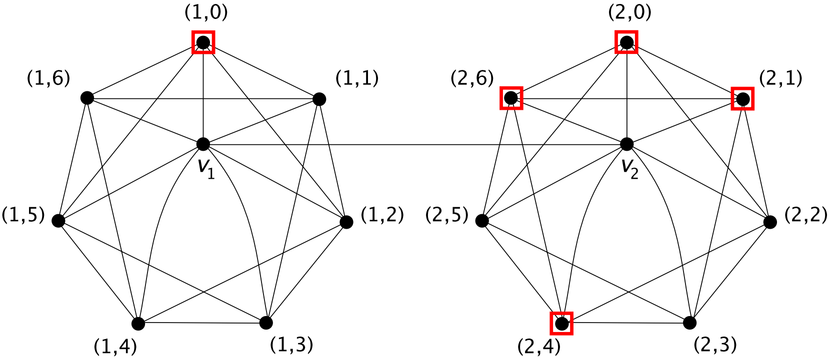
5-Primitive Graph

$n = 16 : G_{16}^{(A)}$



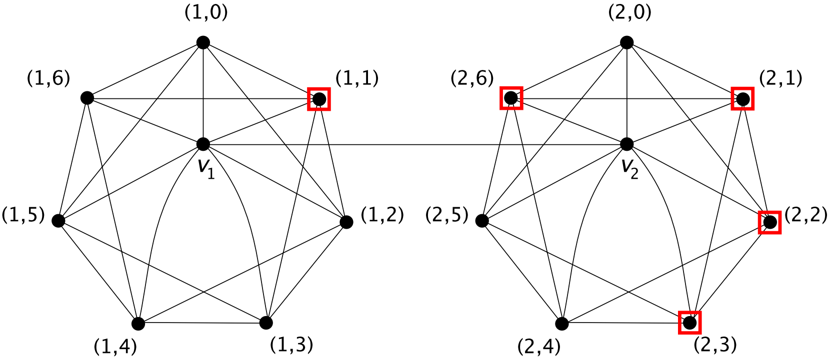
5-Primitive Graph

$n = 16 : G_{16}^{(A)}$



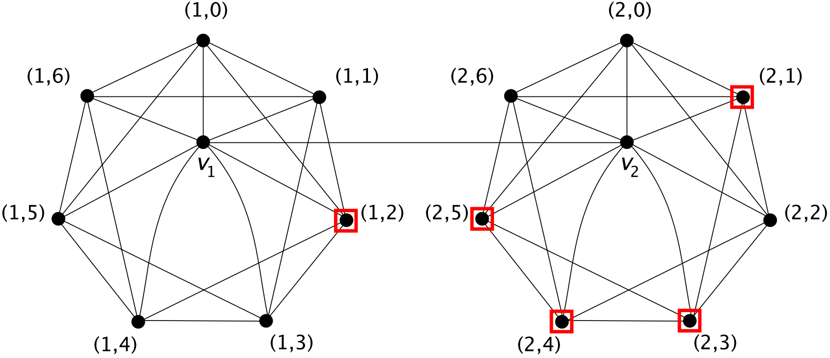
5-Primitive Graph

$n = 16 : G_{16}^{(A)}$



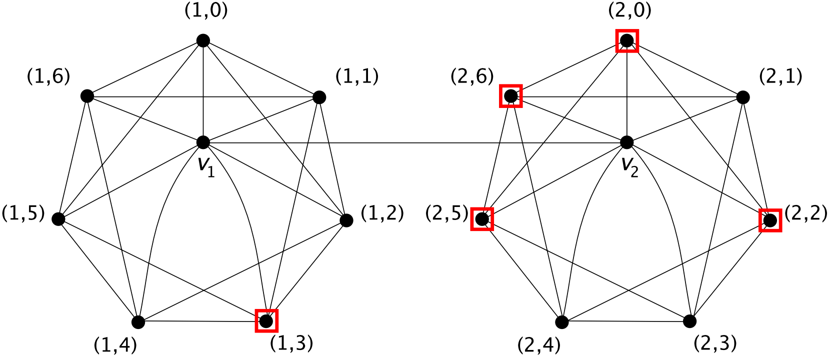
5-Primitive Graph

$n = 16 : G_{16}^{(A)}$



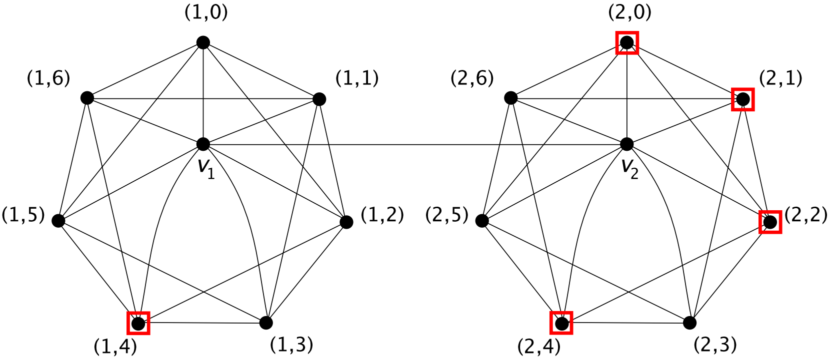
5-Primitive Graph

$n = 16 : G_{16}^{(A)}$



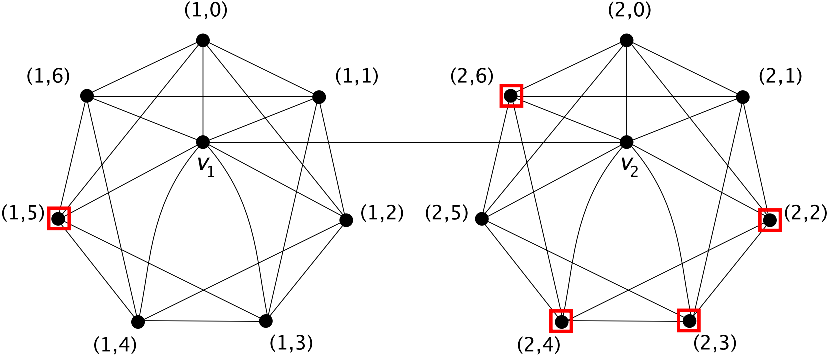
5-Primitive Graph

$n = 16 : G_{16}^{(A)}$



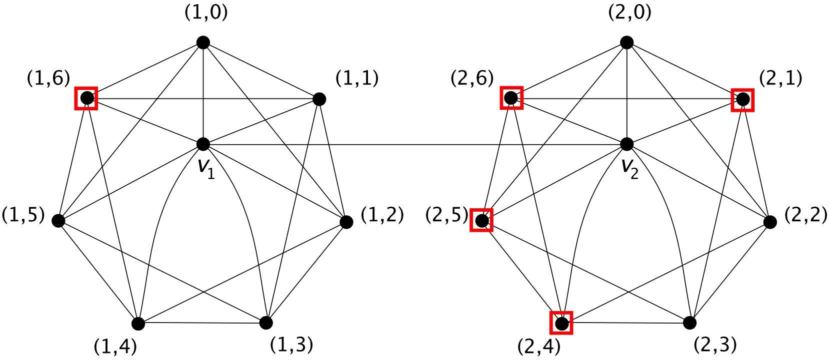
5-Primitive Graph

$n = 16 : G_{16}^{(A)}$



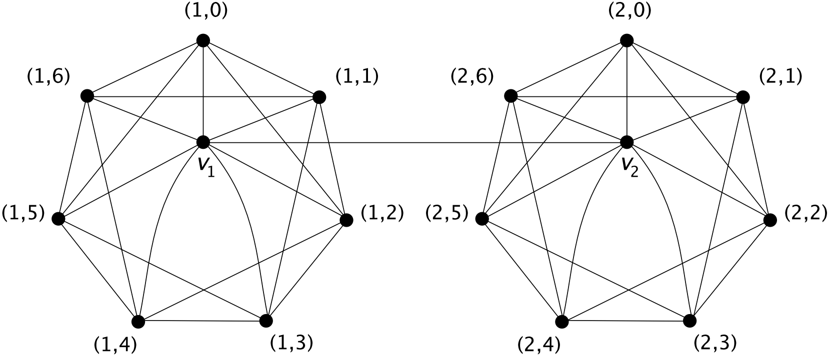
5-Primitive Graph

$n = 16 : G_{16}^{(A)}$



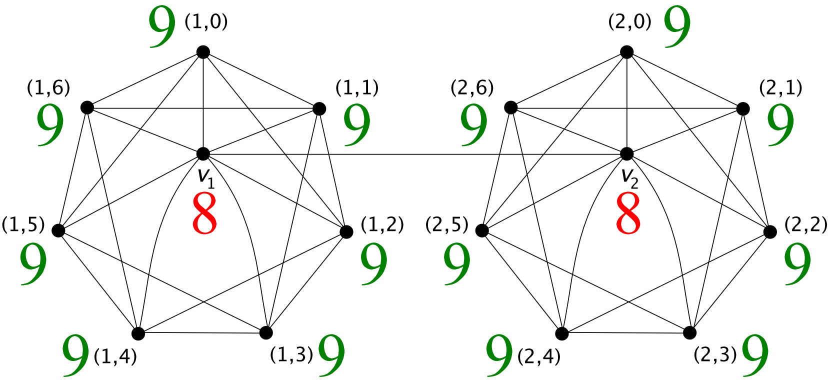
5-Primitive Graph

$n = 16 : G_{16}^{(A)}$

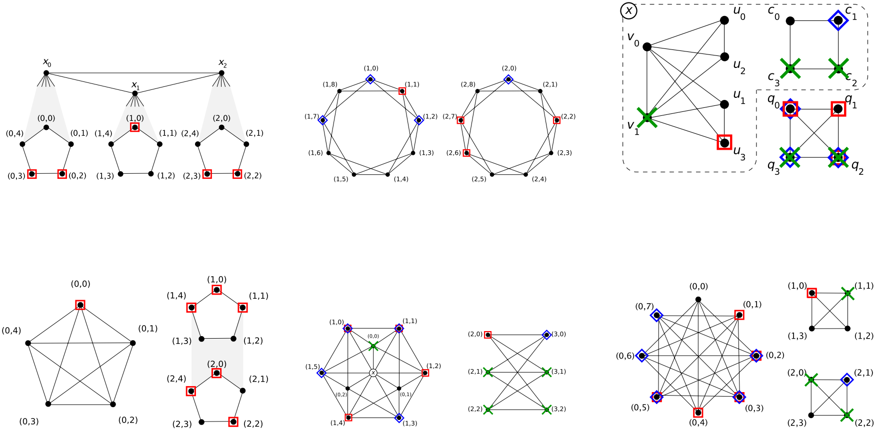


5-Primitive Graph

$n = 16 : G_{16}^{(A)}$

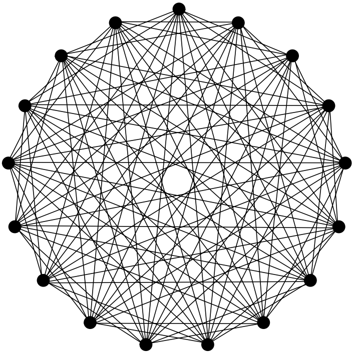


Other r-Primitive Graphs ($r \in \{4, 5, 6\}$)



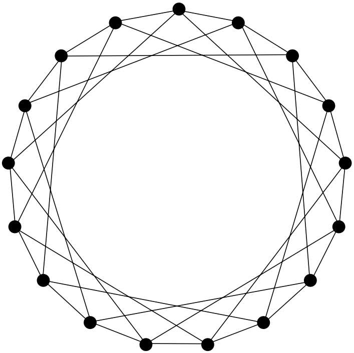
7-Primitive Graph

$n = 17 : G_{17}^{(A)}$



7-Primitive Graph

$n = 17 : G_{17}^{(A)}$



Let Γ be a group and $S \subseteq \Gamma$ a set of generators.

The undirected **Cayley graph** $C(\Gamma, S)$ has vertex set Γ and for all $a \in \Gamma$ and $b \in S$, there is an edge between a and ab.

Let Γ be a group and $S \subseteq \Gamma$ a set of generators.

The undirected **Cayley graph** $C(\Gamma, S)$ has vertex set Γ and for all $a \in \Gamma$ and $b \in S$, there is an edge between a and ab.

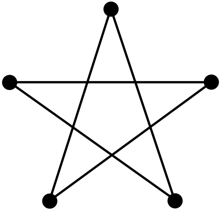
The **Cayley complement** $\overline{C}(\Gamma, S)$ is the complement of $C(\Gamma, S)$.

Let Γ be a group and $S \subseteq \Gamma$ a set of generators.

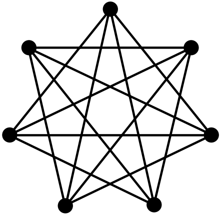
The undirected **Cayley graph** $C(\Gamma, S)$ has vertex set Γ and for all $a \in \Gamma$ and $b \in S$, there is an edge between a and ab.

The **Cayley complement** $\overline{C}(\Gamma, S)$ is the complement of $C(\Gamma, S)$.

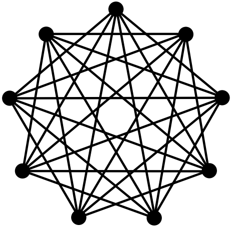
For $r \geq 1$, $\overline{C}(\mathbb{Z}_{2r-1}, \{1\}) \cong \overline{C_{2r-1}}$ is r-primitive.



$\overline{C_5}$



$\overline{C_7}$



$\overline{C_9}$

Searching for r-Primitive Cayley Complements

To search for Cayley complements $\overline{C}(\mathbb{Z}_n, S)$ with $|S| = g$:

1. Select a generator set $S = \{a_1 = 1 < a_2 < a_3 < \cdots < a_g\}$.

Searching for r-Primitive Cayley Complements

To search for Cayley complements $\overline{C}(\mathbb{Z}_n, S)$ with $|S| = g$:

1. Select a generator set $S = \{a_1 = 1 < a_2 < a_3 < \cdots < a_g\}$.
2. Select an integer $n > 2a_g$.

Searching for r-Primitive Cayley Complements

To search for Cayley complements $\overline{C}(\mathbb{Z}_n, S)$ with $|S| = g$:

1. Select a generator set $S = \{a_1 = 1 < a_2 < a_3 < \cdots < a_g\}$.
2. Select an integer $n > 2a_g$.
3. Compute $r = \omega(\overline{C}(\mathbb{Z}_n, S)) + 1$.

Searching for r-Primitive Cayley Complements

To search for Cayley complements $\overline{C}(\mathbb{Z}_n, S)$ with $|S| = g$:

1. Select a generator set $S = \{a_1 = 1 < a_2 < a_3 < \cdots < a_g\}$.
2. Select an integer $n > 2a_g$.
3. Compute $r = \omega(\overline{C}(\mathbb{Z}_n, S)) + 1$.
4. Check if $\overline{C}(\mathbb{Z}_n, S) + \{0, a_i\}$ has a unique r-clique for all $a_i \in S$.

Searching for r-Primitive Cayley Complements

To search for Cayley complements $\overline{C}(\mathbb{Z}_n, S)$ with $|S| = g$:

1. Select a generator set $S = \{a_1 = 1 < a_2 < a_3 < \cdots < a_g\}$.
2. Select an integer $n > 2a_g$.
3. Compute $r = \omega(\overline{C}(\mathbb{Z}_n, S)) + 1$.
4. Check if $\overline{C}(\mathbb{Z}_n, S) + \{0, a_i\}$ has a unique r-clique for all $a_i \in S$.

Implementation uses Niskanen and Östergård's *cliquer* software to compute $\omega(G)$.

Two or Three Generators

S	r	n
$\{1,4\}$	7	17
$\{1,6\}$	16	37
$\{1,8\}$	29	65
$\{1,10\}$	46	101
$\{1,12\}$	67	145

$g = 2$

S	r	n
$\{1,5,6\}$	9	31
$\{1,8,9\}$	22	73
$\{1,11,12\}$	41	133
$\{1,14,15\}$	66	211
$\{1,17,18\}$	97	307

$g = 3$

Infinite Families

Conjecture (Hartke, Stolee, 2012) Let $t \geq 1$,

$$n = 4t^2 + 1, \qquad \text{and} \qquad r = 2t^2 - t + 1.$$

The Cayley complement $\overline{C}(\mathbb{Z}_n, \{1, 2t\})$ is r-primitive.

Conjecture (Hartke, Stolee, 2012) Let $t \geq 1$,

$$n = 9t^2 - 3t + 1 \qquad \text{and} \qquad r = 3t^2 - 2t + 1.$$

The Cayley complement $\overline{C}(\mathbb{Z}_n, \{1, 3t-1, 3t\})$ is r-primitive.

Infinite Families

Theorem (Hartke, Stolee, 2012) Let $t \geq 1$,

$$n = 4t^2 + 1, \qquad \text{and} \qquad r = 2t^2 - t + 1.$$

The Cayley complement $\overline{C}(\mathbb{Z}_n, \{1, 2t\})$ is r-primitive.

Proof uses **counting** method.

Conjecture (Hartke, Stolee, 2012) Let $t \geq 1$,

$$n = 9t^2 - 3t + 1 \qquad \text{and} \qquad r = 3t^2 - 2t + 1.$$

The Cayley complement $\overline{C}(\mathbb{Z}_n, \{1, 3t-1, 3t\})$ is r-primitive.

Infinite Families

Theorem (Hartke, Stolee, 2012) Let $t \geq 1$,

$$n = 4t^2 + 1, \qquad \text{and} \qquad r = 2t^2 - t + 1.$$

The Cayley complement $\overline{C}(\mathbb{Z}_n, \{1, 2t\})$ is r-primitive.

Proof uses **counting** method.

Theorem (Hartke, Stolee, 2012) Let $t \geq 1$,

$$n = 9t^2 - 3t + 1 \qquad \text{and} \qquad r = 3t^2 - 2t + 1.$$

The Cayley complement $\overline{C}(\mathbb{Z}_n, \{1, 3t-1, 3t\})$ is r-primitive.

Proof uses **discharging** method.

More Experimental Results

Our search for r-primitive Cayley complements also found these constructions:

g	S	r	n
4	$\{1, 5, 8, 34\}$ $\{1, 11, 18, 34\}$	28	89
5	$\{1, 5, 14, 17, 25\}$	19	71
5	$\{1, 6, 14, 17, 36\}$	27	101
6	$\{1, 6, 16, 22, 35, 36\}$	21	97
7	$\{1, 20, 23, 26, 30, 32, 34\}$	15	71
8	$\{1, 8, 12, 18, 22, 27, 33, 47\}$	20	97

It remains to be seen if these extend to infinite families.

Open Questions

Open Questions

1. Is there an **infinite** family of r-primitive graphs for a **fixed** r?

Open Questions

1. Is there an **infinite** family of r-primitive graphs for a **fixed** r?

2. Can one bound $\Delta(G)$ and/or $\delta(G)$ for r-primitive G?

Open Questions

1. Is there an **infinite** family of r-primitive graphs for a **fixed** r?

2. Can one bound $\Delta(G)$ and/or $\delta(G)$ for r-primitive G?

3. Is there an infinite family of **irregular** r-primitive graphs?
   Can $\Delta(G) - \delta(G)$ become **arbitrarily large**?

Open Questions

1. Is there an **infinite** family of r-primitive graphs for a **fixed** r?
2. Can one bound $\Delta(G)$ and/or $\delta(G)$ for r-primitive G?
3. Is there an infinite family of **irregular** r-primitive graphs? Can $\Delta(G) - \delta(G)$ become **arbitrarily large**?
4. Is $\overline{C}(\Gamma, S)$ r-primitive for any group $\Gamma \ncong \mathbb{Z}_n$?

Uniquely K_r-Saturated Graphs

Derrick Stolee

Joint with Stephen G. Hartke

University of Nebraska–Lincoln
s-dstolee1@math.unl.edu
http://www.math.unl.edu/∼s-dstolee1/

January 6, 2012

Supported by NSF grant DMS-0914815 and a University of Nebraska Presidential Fellowship.