MATH 412, FALL 2012 - HOMEWORK 1

WARMUP PROBLEMS: Section 1.1: #2, 4, 5, 7, 8, 9. Section 1.2: #1, 2, 5. Do not write these up! Just think about how to solve them to make sure you understand the material before working on the written homework.

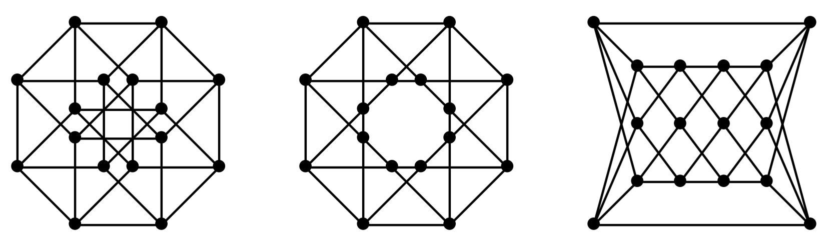
EXTRA PROBLEMS: Section 1.1: #11, 13, 14, 18, 19, 24, 27, 31. Section 1.2: #14, 17, 18, 20. Do not write these up! If you have time after doing the homework, think about these for extra practice.

WRITTEN PROBLEMS: Solve five of the following six problems (students registered for four credit hours or honors must do all six). Due Wednesday, Sept. 5. Problem sets will usually be due on Wednesdays, with solution sets distributed on Fridays and graded homework returned on Mondays. Some problems have hints in the back of the book; try them first without the hints. Come to the collaborative study sessions or office hours if you have trouble.

Words like "construct", "show", "obtain", "determine", etc., explicitly state that proof is required. Full credit for solutions to most problems requires proof of the statements made. Use *sentences*; you cannot give a proof without words. Results covered in class can be used without proof if stated correctly.

1. Determine whether the Petersen graph is bipartite, and find the size of its largest independent set.

2. Prove that no two graphs below are isomorphic.



3. *The Odd Graph O_k.* The vertices of the graph O_k are the k-element subsets of $\{1, 2, \ldots, 2k+1\}$. Two vertices are adjacent if and only if they are disjoint sets. Thus O_2 is the Petersen graph. Prove that the girth of O_k is 6 if $k \geq 3$.

4. Let G be a simple graph in which every vertex has degree 3. Prove that G decomposes into claws if and only if G is bipartite.

5. Let e be an edge appearing an odd number of times in a closed walk W. Prove that W contains the edges of a cycle through e.

6. Let G be a self-complementary graph. Prove that G has a cut-vertex if and only if G has a vertex of degree 1.