# MATH 412, FALL 2012 - HOMEWORK 3

WARMUP PROBLEMS: Section 1.3 #8, 46. Section 1.4 #1, 3, 4, 5, 8, 10. Do not write these up! Use these to clarify your understanding.

OTHER INTERESTING PROBLEMS: Section 1.3 #20, 25, 31, 40, 41 45, 47, 49, 51–53, 57, 61, 64. Section 1.4 #11, 14, 20, 21, 23, 25, 28, 29, 32, 36, 37, 40. Do not write these up! Think about some if you have time.

WRITTEN PROBLEMS: Solve and write up five of the following six (students registered for four credits or honors do all six problems). Due Wednesday, September 19.

COMMENT: When using induction to prove an implication, keep the template of Remark 1.3.25 in mind. Induction is a promising technique when a current instance of the problem can be solved by using a solution to a smaller instance.

**1.** For $k \geq 2$, prove that every $k$-regular bipartite graph has no cut-edge, and construct a bipartite graph with a cut-edge whose vertex degrees all lie in $\{k, k+1\}$.

**2.** Count the 6-cycles in $Q_3$. Prove that every 6-cycle in $Q_k$ lies in exactly one 3-dimensional subcube. Use this to count the 6-cycles in $Q_k$ for $k \geq 3$.

**3.** For each $k \in \mathbb{N}$ and each loopless graph $G$, prove that $G$ has a $k$-partite subgraph $H$ (Definition 1.1.12) such that $e(H) \geq (1 - 1/k)e(G)$.

**4.** Define $d = (d_1, \ldots, d_{2k})$ by $d_{2i} = d_{2i-1} = i$ for $1 \leq i \leq k$. Prove that $d$ is graphic. (Hint: Do not use the Havel–Hakimi Theorem.)

**5.** *DeBruijn sequence for any alphabet and length.* Let $A$ be an alphabet of size $k$. Prove that there exists a cyclic arrangement of $k^l$ characters chosen from $A$ such that the $k^l$ strings of length $l$ in the sequence are all distinct.

**6.** Let $p_1, \ldots, p_n$ be nonnegative integers with $p_1 \leq \cdots \leq p_n$. Let $p'_k = \sum_{i=1}^{k} p_i$. Prove that there exists a tournament with outdegrees $p_1, \ldots, p_n$ if and only if $p'_k \geq \binom{k}{2}$ for $1 \leq k < n$ and $p'_n = \binom{n}{2}$. (Hint: Use induction on $\sum_{k=1}^{n}[p'_k - \binom{k}{2}]$.)