MATH 412, FALL 2012 - HOMEWORK 4

WARMUP PROBLEMS: Section 2.1 #3, 4, 8, 10, 11. Section 2.2 #1, 2, 4. Do not write these up!

EXTRA PROBLEMS: Section 2.1 #17, 19, 21, 23, 27, 47. Section 2.2 #6, 10, 12, 20, 38. Do not write these up!

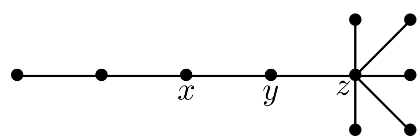
WRITTEN PROBLEMS: Solve and write up five of the following six (students registered for four credits or honors do all six problems). Due Wednesday, September 26.

1. Prove that among trees with n vertices, the star has the most independent sets.

2. Given $x \in V(G)$, let $s(x) = \sum_{v \in V(G)} d(x, v)$. The *barycenter* of G is the set of vertices at which $s(x)$ is minimized (also called the *median*).

a) Prove that the barycenter of a tree is a single vertex or two adjacent vertices. (Hint: Study $s(u) - s(v)$ when u and v are adjacent.)

b) Determine the maximum distance between the center and the barycenter in a tree of diameter d. (Example: In the tree below, the center is $\{x, y\}$, the barycenter is $\{z\}$, and the distance between them is 1.)



3. Let G be a connected n-vertex graph with minimum degree k, where $n - 3 \geq k \geq 2$. Prove that $\operatorname{diam} G \leq 3(n-2)/(k+1) - 1$. Prove that if $k+1$ divides $n-2$, then there is a graph where the bound holds with equality.

4. Let G be an n-vertex simple graph that decomposes into k spanning trees. Given also that $\Delta(G) = \delta(G) + 1$, determine the degree sequence of G in terms of n and k.

5. Use the Matrix Tree Theorem to prove Cayley's Formula.

6. Prove that if a graph G is graceful and Eulerian, then $e(G)$ is congruent to 0 or 3 mod 4. (Hint: Sum the absolute edge differences (mod 2) in two different ways.)