MATH 412, FALL 2012 - HOMEWORK 13

WARMUP PROBLEMS: Section 6.3 #1, 3, 4, 16. Section 7.1 #1, 2, 4, 5, 6. Do not write these up!

EXTRA PROBLEMS: Section 6.3: #5, 6, 10, 11, 15, 17, 21, 24, 26, 30. Section 7.1: #9, 10, 11, 12, 14, 17, 18, 19, 22, 26. Do not write these up!

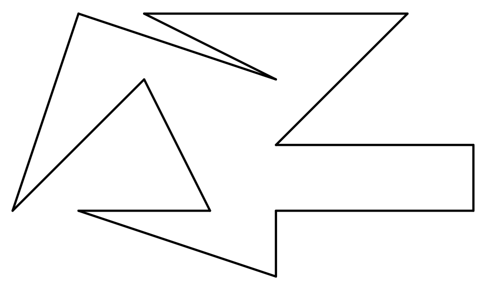
WRITTEN PROBLEMS: Do five of the following six. Due Wednesday, December 5.

1. *Short proof of the Five Color Theorem.*

a) Let v be a 5-vertex in a plane graph G. Let x and y be nonadjacent neighbors of v, and let G' be the graph obtained from G by contracting the edges vx and vy. Prove that if G' is 5-colorable, then G is 5-colorable.

b) Use part (a) to give a short inductive proof of the Five Color Theorem.

2. Without using the Four Color Theorem, prove that every outerplanar graph is 3-colorable. Apply this to prove the Art Gallery Theorem: If an art gallery is laid out as a simple polygon with n sides, then it is possible to place $\lfloor n/3 \rfloor$ guards such that every point of the interior is visible to some guard. For $n \geq 3$, construct a polygon that requires $\lfloor n/3 \rfloor$ guards.



3. The *thickness* of a graph G is the minimum number of planar graphs needed to partition $E(G)$.

a) Prove that if G has thickness 2, then $\chi(G) \leq 12$.

b) For r even and s greater than $(r-2)^2/2$, prove that the thickness of $K_{r,s}$ is $r/2$.

4. Suppose that m and n are odd. Prove that in all drawings of $K_{m,n}$, the parity of the number of pairs of nonincident edges that cross an odd number of times is the same. Conclude that $\nu(K_{m,n})$ is odd when $m-3$ and $n-3$ are divisible by 4 and even otherwise.

5. Use Tutte's 1-factor Theorem to prove that every connected line graph of even order has a perfect matching. Conclude from this that every simple connected graph of even size decomposes into paths of length 2. (Comment: Exercise 3.3.23 shows that every connected claw-free graph has a perfect matching; that stronger result is more difficult than this.)

6. *Density conditions for $\chi'(G) > \Delta(G)$.*

a) Prove that if $n(G) = 2m+1$ and $e(G) > m \cdot \Delta(G)$, then $\chi'(G) > \Delta(G)$.

b) Prove that if G is obtained from a k-regular graph with $2m+1$ vertices by deleting fewer than $k/2$ edges, then $\chi'(G) > \Delta(G)$.

c) Prove that if G is obtained by subdividing an edge of a regular graph with $2m$ vertices and degree at least 2, then $\chi'(G) > \Delta(G)$.