MATH 412, SPRING 2013 - HOMEWORK 1

WARMUP PROBLEMS: Section 1.1: #2, 4, 5, 7, 8, 9. Section 1.2: #1, 2, 5. Do not write these up! Just think about how to solve them to make sure you understand the material before working on the written homework.

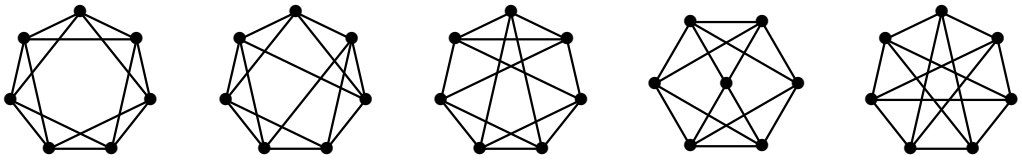
EXTRA PROBLEMS: Section 1.1: #11, 12, 14, 19, 21, 24, 25, 28, 31. Section 1.2: #14, 16, 17, 18, 21. Do not write these up! If you have time after doing the homework, think about these for extra practice.

WRITTEN PROBLEMS: Solve five of the following six problems (students registered for four credit hours or honors must do all six). Due Wednesday, Jan. 23. Problem sets will usually be due on Wednesdays, with solution sets distributed on Fridays and graded homework returned on Mondays. Some problems have hints in the back of the book; try them first without the hints. Come to the collaborative study sessions or office hours if you have trouble.

Words like "construct", "show", "obtain", "determine", etc., explicitly state that proof is required. Full credit for solutions to most problems requires proof of the statements made. Use *sentences*; you cannot give a proof without words. Results covered in class can be used without proof if stated correctly.

1. Let G be the graph whose vertex set is the set of k-tuples with coordinates in $\{0, 1\}$, with x adjacent to y when x and y differ in exactly one position. Determine whether G is bipartite.

2. Find the equivalence classes of the isomorphism relation on the set of graphs below.



3. Let G be a graph with girth 5. Prove that if every vertex of G has degree at least k, then G has at least $k^2 + 1$ vertices. For $k = 2$ and $k = 3$, find one such graph with exactly $k^2 + 1$ vertices.

4. Determine which complete bipartite graphs decompose into two isomorphic subgraphs.

5. Let W be a closed walk of length at least 1 that does not contain a cycle. Prove that some edge of W repeats immediately (once in each direction).

6. Let v be a cut-vertex of a simple graph G. Prove that $\overline{G} - v$ is connected.