MATH 412, SPRING 2013 - HOMEWORK 2

WARMUP PROBLEMS: Section 1.2: #3, 6, 8, 10, 12. Section 1.3: #1, 9. Do not write these up! Think about how to solve them to make sure you understand the material before doing the homework.

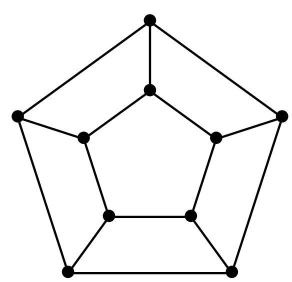
EXTRA PROBLEMS: Section 1.2: #15, 23, 25, 27, 28, 32, 33, 37, 40, 41. Section 1.3: #10, 14. Do not write these up! These are interesting problems (related to what we have discussed) to provide extra practice.

WRITTEN PROBLEMS: Do five of the six problems below (students registered for four hours or honors must do all six problems). Due Wednesday, January 29.

1. Prove that a graph G is bipartite if and only if every subgraph H of G has an independent set consisting of at least half of $V(H)$.

2. Let G be a simple graph with vertices $v_1, \dots, v_n$. Let A^k denote the kth power of the adjacency matrix of G under matrix multiplication. Prove that entry i, j of A^k is the number of v_i, v_j-walks of length k in G. Prove that G is bipartite if and only if, for the odd integer r in $\{n-1, n\}$, the diagonal entries of A^r are all 0. (Reminder: A walk is an *ordered* list of vertices and edges.)

3. Prove that the graph below cannot be expressed as the union of two cycles.



4. Let G be a graph with girth g in which every vertex has degree at least k. Prove that if $k \geq 2$, then G contains a cycle of length at least $(g-2)(k-1)+2$.

5. Use induction on k to prove that every connected simple graph with $2k$ edges decomposes into paths of length 2. Does the conclusion remain true without the hypothesis of connectedness?

6. Let W be a closed walk in a graph G. Let H be the subgraph of G consisting of edges used an odd number of times in W. Prove that $d_H(v)$ is even for every $v \in V(G)$.