MATH 412, SPRING 2013 - HOMEWORK 11

WARMUP PROBLEMS: Section 5.2 #3, 4, 5. Section 6.1 #1–5, 7, 8, 11.

EXTRA PROBLEMS: Section 5.2 #25, 26, 29, 31, 32, 37, 40. Section 6.1 #13, 14, 15, 17, 20, 21, 23, 24, 27, 33.

WRITTEN HOMEWORK: Do five of the following six. Due Wed., April 10.

1. Let G be a 4-critical graph having a separating set S of size 4. Prove that $G[S]$ has at most four edges.

2. K_4-*subdivisions.*

a) Prove that every simple graph with minimum degree at least 3 contains a K_4-subdivision. (Hint: Prove the stronger result that every nontrivial simple graph with at most one vertex of degree less than 3 contains a K_4-subdivision. The proof of Theorem 5.2.20 already shows that every 3-connected graph contains a K_4-subdivision.)

b) Conclude from part (a) that if $n(G) \geq 3$ and G does not contain a K_4-subdivision, then $e(G) \leq 2n(G) - 3$. Give a construction to show that the bound is best possible.

3. Prove that every n-vertex plane graph isomorphic to its dual has $2n - 2$ edges. For all $n \geq 4$, construct a simple n-vertex plane graph isomorphic to its dual.

4. Let G be an n-vertex simple planar graph with girth k, where k is finite. Prove that G has at most $(n-2)\frac{k}{k-2}$ edges. Use this to prove that the Petersen graph is nonplanar.

5. Let ℓ be the length of a longest cycle in a planar triangulation G. Prove that G has cycles of all lengths from 3 through ℓ.

6. Prove that every simple planar graph with at least four vertices has at least four vertices with degree less than 6. For each even value of n with $n \geq 8$, construct an n-vertex simple planar graph G that has exactly four vertices with degree less than 6.