## First example of simplex method

Suppose we are given the problem

Subject to  

$$\begin{cases}
x_1 - 2x_2 + x_3 + x_4 = 10 \\
2x_1 - 3x_2 - x_3 + x_5 = 6 \\
x_1, x_2, x_3, x_4, x_5 \ge 0.
\end{cases}$$

Rewrite the objective function as a new equation with the new variable z and switch the LHS with the RHS.

$$\begin{cases} 0 = -z - x_1 + 2x_2 - x_3 \\ 10 = x_1 - 2x_2 + x_3 + x_4 \\ 6 = 2x_1 - 3x_2 - x_3 + x_5 \\ x_1, x_2, x_3, x_4, x_5 \ge 0. \end{cases}$$

A good thing about this system is that it is solved with respect to  $x_4$  and  $x_5$ . Introduce the new variable -z and consider the tableau consisting of the coefficients of the system:

		<b>x</b> <sub>1</sub>	$x_2$	$x_3$	$x_4$	$x_5$
-z	0	-1	2	-1	0	0
$x_4$	10	1	-2	1	1	0
$x_5$	6	2	-3	-1	0	1

The variables on the left indicate with respect to which variable the corresponding equation is solved. Note that this tableau corresponds to the basic solution  $x_1 = x_2 = x_3 = 0$ ,  $x_4 = 10$ ,  $x_5 = 6$  which gives -z = 0.

Check the 0th row whether there are negative entries in columns distinct from 0th. There are negative coefficients at  $x_1$  and  $x_3$ . Choose, for example  $x_1$  (it is in bold in the picture). The corresponding column will be the *pivot column*. Both entries in this column are positive. The ratio 10/1 is greater than the ratio 6/2, so the *pivot row* will be Row 2, and 2 will be the *pivot entry*. Pivoting (excluding  $x_1$  from all other equations), we come to the tableau

		$x_1$	$x_2$	$x_3$	$x_4$	$x_5$
-z	3	0	1/2	-3/2	0	1/2
$x_4$	7	0	-1/2	3/2	1	-1/2
$x_1$	3	1	-3/2	-1/2	0	1/2

This tableau corresponds to the basic solution  $x_5 = x_2 = x_3 = 0$ ,  $x_1 = 3$ ,  $x_4 = 7$  which gives -z = 3. So, this solution is better than the previous. Now, the only negative entry in the 0th row is the coefficient at  $x_3$ . So, this is the pivot column. In this column, there is only one positive entry, so we pivot on this entry:

		$x_1$	$x_2$	$x_3$	$x_4$	$x_5$
-z	10	0	0	0	1	0
$x_3$	14/3	0	-1/3	1	2/3	-1/3
$x_1$	16/3	1	-5/3	0	1/3	1/3

This tableau corresponds to the basic solution  $x_5 = x_2 = x_4 = 0$ ,  $x_1 = 16/3$ ,  $x_3 = 14/3$ , which gives -z = 10. This solution not only is better than the previous, but since we do not have negative entries in Row 0, this solution is optimal (!!). Indeed, the 0th row reads as the following equation:

$$z = -10 + x_4.$$

Since  $x_4 \ge 0$ , z never can be less than -10 and we are given a solution with z = -10.