MATH 482, Spring 2013 - Homework 1
Due Wednesday, 01/30.

Solve 4 of the first 5 problems below, and also solve problem 6. Students registered for 4 credits must solve all problems.

1. [5pts] A paper recycling machine can produce toilet paper, writing pads, and paper towels, which sell for 18, 29 and 25 cents and consume 0.5, 0.22 and 0.75 kilograms of newspaper and 0.2, 0.4, and 0.22 minutes. Each day 10 hours and 1500 kilograms of newspaper are available, and at least 1000 rolls of toilet paper, 200 writing pads and 400 rolls of paper towels are required. Formulate an appropriate LP to maximize revenue.

2. [5pts] You have 100 quarters and 90 dimes and no other money. You have to pay a given amount C. No change is given to you. You want to minimize your overpay. State this problem in mathematical terms. Is this problem linear? Solve it for $C = 15$ cents, for $C =\$1.02$, and for $C =\$50$.

3. [5pts] Solve the problem:

$$\max 6x_1 + x_2$$

with respect to

$$
\begin{array}{rrcl}
4x_1 & +5x_2 & \leq & 20, \\
3x_1 & +x_2 & \leq & 6, \\
& x_1, x_2 & \geq & 0.
\end{array}
$$

4. [5pts] State in the canonical form:

$$\min 2x_1 - 3x_2 + x_3$$

with respect to

$$
\begin{array}{rrrrcl}
3x_1 & -2x_2 & +x_3 & & \leq & -5, \\
-x_1 & -3x_2 & +4x_3 & & \leq & -9, \\
x_1 & +x_2 & & +x_4 & = & 6, \\
& x_1, & x_3 & & \geq & 0.
\end{array}
$$

5. [5pts] Consider the constraints

$$
\begin{array}{rrrrrcl}
4x_1 & +2x_2 & -3x_3 & & & = & 4 \\
& x_2 & +3x_3 & +x_4 & & = & 2 \\
-x_1 & & +x_3 & & +4x_5 & = & -3. \\
x_1, & x_2, & x_3, & x_4, & x_5 & \geq & 0
\end{array}
$$

List all basic feasible solutions for these constraints.

6. [10pts] (*You MUST do this problem!*) Solve the following LP using the simplex algorithm with tableaus.

$$
\begin{array}{rrrrcr}
\min & -60x_1 & -90x_2 & -300x_3 & & \\
\text{subject to} & x_1 & +x_2 & +x_3 & \leq & 600 \\
& x_1 & +3x_2 & & \leq & 600 \\
& 2x_1 & & +x_3 & \leq & 900 \\
& x_1, & x_2, & x_3 & \geq & 0
\end{array}
$$