MATH 482, Spring 2013 - Homework 2
Due Wednesday, 02/13.

Solve 4 of the first 5 problems below, and also solve problem 6. Students registered for 4 credits must solve all problems.

1. [5pts] Check whether the vector $[3, -1, 0, 2]$ is an optimal solution to the problem

$$\max 6x_1 + x_2 - x_3 - x_4$$

subject to

$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 & \leq & 5, \\ 3x_1 + x_2 - x_3 & \leq & 8, \\ x_2 + x_3 + x_4 & = & 1, \\ x_3, x_4 & \geq & 0. \end{cases}$$

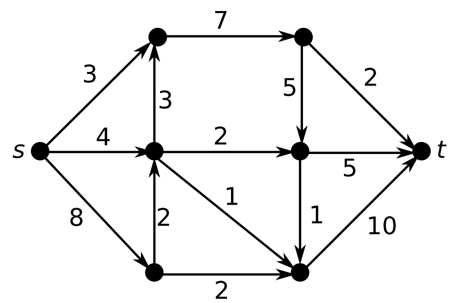
2. [5pts] Solve with two-phase simplex algorithm the LP represented by the tableau below.

	x_1	x_2	x_3	x_4	x_5
0	-3	-4	0	0	0
6	2	1	1	0	0
2	1	-2	0	1	0
1	-3	0	9	9	1

The second row (starting 0) corresponds to the cost vector.

3. [5pts] Prove the theorem due to P. Gordan (1873) that the system $\mathbf{Ax} < \mathbf{0}$ is unsolvable if and only if the system $\mathbf{y}^{\mathbf{T}}\mathbf{A} = \mathbf{0}$, $\mathbf{y} \geq \mathbf{0}$, $\mathbf{y} \neq \mathbf{0}$ is solvable. (Hint: In order to apply duality theorems, replace the system $\mathbf{Ax} < \mathbf{0}$ of strict inequalities by a system of nonstrict inequalities that is solvable if and only if $\mathbf{Ax} < \mathbf{0}$ is solvable).

4. [5pts] Consider the network below.



Write the LP for the shortest-paths problem and the dual problem.

5. [5pts] Demonstrate an optimal solution to the primal LP in Problem 4 and use duality to prove optimality.

6. [10pts] Use the dual simplex method to find an optimal solution to the problem

$$\text{Minimize } z = 7x_1 + x_2 + 3x_3 + x_4$$

subject to

$$\begin{cases} 2x_1 - 3x_2 - x_3 + x_4 & \geq & 8, \\ 6x_1 + x_2 + 2x_3 - 2x_4 & \geq & 12, \\ -x_1 + x_2 + x_3 + x_4 & \geq & 10, \\ x_1, x_2, x_3, x_4 & \geq & 0. \end{cases}$$

The additional constraints

$$\begin{array}{rcl} x_1 + 5x_2 + x_3 + 7x_4 & \leq & 50, \\ 3x_1 + 2x_2 - 2x_3 - x_4 & \leq & 20 \end{array}$$

are added to those of Problem 6.a. Solve the new problem starting from the previous optimal tableau.