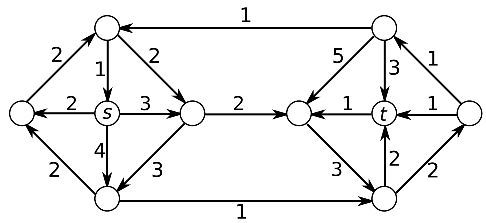
MATH 482, Spring 2013 - Homework 4
Due Wednesday, 03/27.

Solve 4 of the first 5 problems below, and also solve problem 6. Students registered for 4 credits must solve all problems.
For problems 1–3, consider the following graph.



1.[5pts] (*Primal-Dual Simplex*) Solve the shortest st-path problem using the primal-dual simplex algorithm.

2.[5pts] (*Dijkstra's Algorithm*) Use Dijkstra's Algorithm to compute the distances from s to all other vertices in the graph above.

3.[5pts] (*Max-Flow/Min-Cut*) Solve the max-st-flow problem using the Ford-Fulkerson Algorithm. Prove optimality using duality.

4.[5pts] (*Floyd-Warshall Algorithm*) Use the Floyd-Warshall Algorithm to compute shortest distances among all pairs of vertices in the graph given by the following adjacency matrix. (The entry $a_{i,j}$ stores the length of the arc (i, j).)

$$\begin{bmatrix} 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 6 & 1 & 0 & 0 & 3 \\ 5 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \end{bmatrix}$$

5.[5pts] Consider the following linear program.

$$\begin{array}{rrcrcr} \max & x_1 & + & x_2 & & \\ \text{subject to} & -\frac{8}{3}x_1 & + & x_2 & \geq & -\frac{8}{3} \\ & x_1 & - & x_2 & \geq & -\frac{1}{2} \\ & x_1, & & x_2 & \geq & 0 \end{array}$$

Solve the linear problem graphically, then also solve the problem graphically when x_1 and x_2 are constrainted to be integers, demonstrating a gap between the real and integer solutions.

6.[10pts] (*Matchings and Vertex Covers*) Let G be a graph with edges spanning two sets of vertices, X and Y. A *matching* is a set M of edges, where $M = \{x_i y_i : 1 \leq i \leq k\}$ for some k, $x_i \in X$, and $y_i \in Y$, with $x_i y_i \in E(G)$. A *vertex cover* is a set $Q \subset V(G)$ such that all edges have at least one endpoint in Q. Use Max-Flow/Min-Cut to prove that the maximum size of a matching in a bipartite graph G is equal to the minimum size of a vertex cover. (*Hint:* Add vertices s and t to G, direct the edges, and show that the max st-flow and min st-cut problems are equivalent to the max matching and min vertex cover problems.)