# MATH 482, Spring 2013 - Homework 5
## Due Wednesday, 04/17.

Solve either all of the first 5 problems below, or solve 3 of the first 5 problems and also problem 6. Students registered for 4 credits must solve all problems.

**1.** [5pts] An $(n, k, \lambda, \mu)$ *strongly regular graph* is an undirected graph with vertex set $\{v_1, \ldots, v_n\}$ where every vertex is incident to $k$ edges and for every pair $v_i, v_j$:

- If $v_iv_j$ is an edge, then $v_i$ and $v_j$ have exactly $\lambda$ common neighbors.
- If $v_iv_j$ is not an edge, then $v_i$ and $v_j$ have exactly $\mu$ common neighbors.

For arbitrary $n, k, \lambda, \mu$, construct an integer program encoding the constraints of an $(n, k, \lambda, \mu)$ strongly regular graph. Prove that the feasible integer solutions are in bijection with the $(n, k, \lambda, \mu)$ strongly regular graphs on vertex set $v_1, \ldots, v_n$. (For this last part, we consider the graphs to be *labeled* and so do not worry about isomorphism.)

**2.** [5pts] Let

$$A = \begin{bmatrix} 1 & -1 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}.$$

Determine if $A$ or $B$ is totally unimodular.

**3.** [5pts] Solve the following integer program using the fractional dual algorithm.

$$\begin{array}{rrcrcl} \text{min} & x_1 & + & x_2 & = & z \\ \text{subject to} & x_1 & + & 2x_2 & \geq & 2 \\ & -x_1 & + & 4x_2 & \leq & 3 \\ & x_1, & & x_2 & \geq & 0, \text{ integer} \end{array}$$

**4.** [5pts] Solve the integer program of (3) using branch-and-bound.

**5.** [5pts] Draw plots in the $x_1, x_2$ plane of the feasible regions and cuts at each stage of the fractional dual algorithm or branch-and-bound. You may only do this problem for credit if you do both (3) and (4).

**6.** [10pts] Solve the following integer program using fractional dual or branch-and-bound.

$$\begin{array}{rrcrcrcrcrcrcl} \text{min} & x_1 & + & x_2 & + & x_3 & + & x_4 & + & x_5 & + & x_6 & & \\ \text{subject to} & x_1 & + & x_2 & & & & & & & & & \geq & \frac{5}{3} \\ & x_1 & & & + & x_3 & & & & & & & \geq & \frac{3}{4} \\ & & & x_2 & & & + & x_4 & & & + & x_6 & \geq & \frac{3}{2} \\ & & & & & & & & & x_5 & + & x_6 & \geq & \frac{1}{2} \\ & & & & & x_3 & + & x_4 & + & x_5 & & & \geq & 1 \\ & x_1, & & x_2, & & x_3, & & x_4, & & x_5, & & x_6 & \geq & 0, \text{integer} \end{array}$$