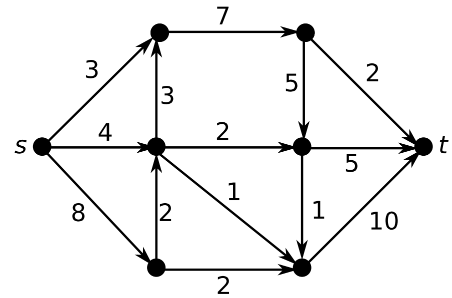
MATH 482, Spring 2013 - Homework 2

Assigned Wednesday 09/18. Due Friday 09/20.

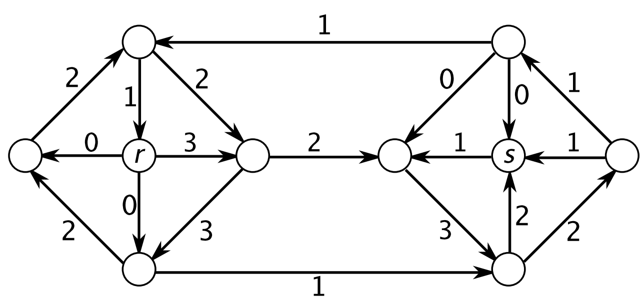
Problems Assigned: 1, 2, 3.a., 4.c.

1. (*Assigned! 5pts*) Consider the graph below.



Find a shortest path and prove optimality using duality.

2. (*Assigned! 5pts*) Consider the network below with given edge values, forming an integer feasible flow.

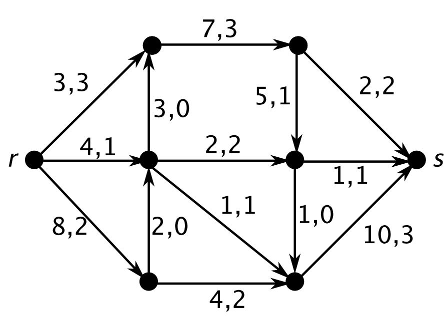


Demonstrate a list of path and cycle flows whose sum is this flow.

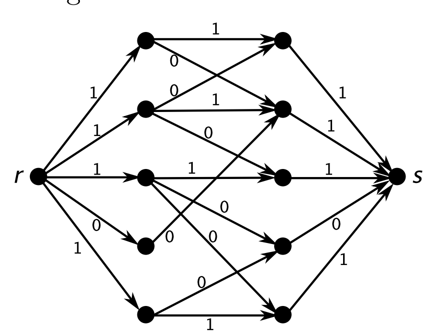
3.

a. (*Assigned! 5pts*) Consider the network below and on the left, with given capacity and flow values. (Recall that an edge label c, x has capacity c and flow-value x.) Find augmenting paths and augment the flow until it has value 8.

b. Consider the network below and on the right, where every edge has capacity 1 and the numbers below are the flow values. Find augmenting paths and augment the flow until it has value 5.

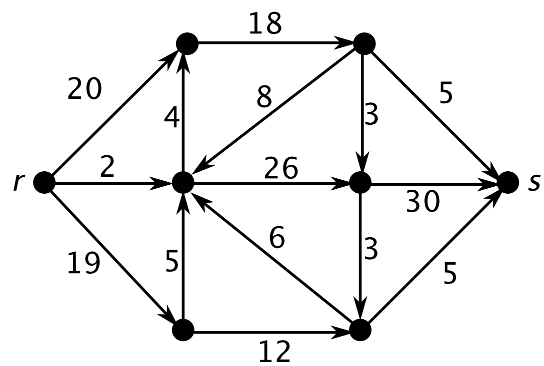


Problem 3.a.



Problem 3.b.

4.
a. Construct a network such that there is a unique maximum flow, but multiple distinct minimum cuts.
b. Construct a network such that there is a unique minimum cut, but multiple distinct maximum flows.
c. (*Assigned!* 5pts) Prove that the network below has a unique maximum flow and a unique minimum cut. (*Hint:* Find a max flow and a min cut. Use the flow to prove there is a unique cut, and use the cut to prove there is a unique flow.)



Note: I did not get to this result in class, so here is the result you should use for 4.c.
Cor: Let $\mathbf{x}$ be a feasible rs-flow and $\delta(R)$ be an rs-cut. Then, $f_{\mathbf{x}}(s) = c(\delta(R))$ if and only if $x_e = c(e)$ for all $e \in \delta(R)$ and $x_e = 0$ for all $e \in \delta(\overline{R})$.