

MATH 482, Spring 2013 - Homework 4
Assigned Monday 10/07. Due Wednesday 10/09.
Assigned: 1.e, 2.d, 3.b, 4.a.

1. Find a maximum-weight perfect matching and a minimum-weight vertex cover for the bipartite graphs with weight matrices given below. (*Assigned: e*)

$$\begin{bmatrix} 6 & 8 & 6 & 3 \\ 3 & 9 & 9 & 7 \\ 5 & 0 & 4 & 1 \\ 0 & 3 & 9 & 7 \end{bmatrix}$$

a.

$$\begin{bmatrix} 3 & 1 & 8 & 5 & 9 \\ 8 & 5 & 1 & 5 & 8 \\ 5 & 6 & 2 & 7 & 1 \\ 3 & 4 & 2 & 3 & 6 \\ 8 & 8 & 3 & 9 & 8 \end{bmatrix}$$

b.

$$\begin{bmatrix} 5 & 7 & 3 & 8 & 0 \\ 4 & 9 & 5 & 8 & 4 \\ 1 & 8 & 9 & 1 & 9 \\ 1 & 7 & 2 & 8 & 2 \\ 9 & 6 & 1 & 1 & 6 \end{bmatrix}$$

c.

$$\begin{bmatrix} 6 & 4 & 4 & 4 & 9 & 9 \\ 5 & 9 & 2 & 1 & 4 & 7 \\ 1 & 0 & 9 & 6 & 3 & 9 \\ 9 & 5 & 4 & 6 & 7 & 9 \\ 5 & 8 & 5 & 1 & 4 & 9 \\ 5 & 5 & 8 & 9 & 7 & 8 \end{bmatrix}$$

d.

$$\begin{bmatrix} 8 & 1 & 3 & 9 & 4 & 3 \\ 3 & 9 & 0 & 8 & 8 & 4 \\ 9 & 1 & 4 & 1 & 0 & 0 \\ 8 & 3 & 5 & 8 & 6 & 1 \\ 6 & 1 & 0 & 0 & 5 & 0 \\ 2 & 1 & 0 & 7 & 0 & 9 \end{bmatrix}$$

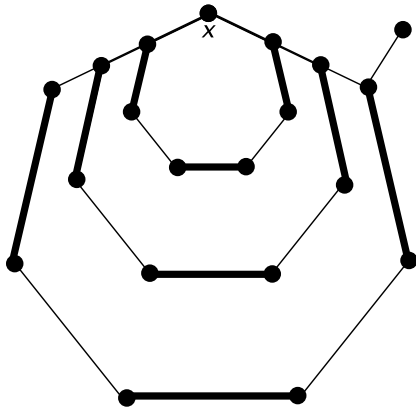
e.

$$\begin{bmatrix} 6 & 3 & 4 & 4 & 1 & 6 \\ 7 & 2 & 8 & 7 & 5 & 9 \\ 2 & 7 & 3 & 3 & 2 & 1 \\ 9 & 0 & 1 & 0 & 7 & 8 \\ 7 & 0 & 0 & 6 & 6 & 5 \\ 9 & 6 & 9 & 6 & 7 & 6 \end{bmatrix}$$

f.

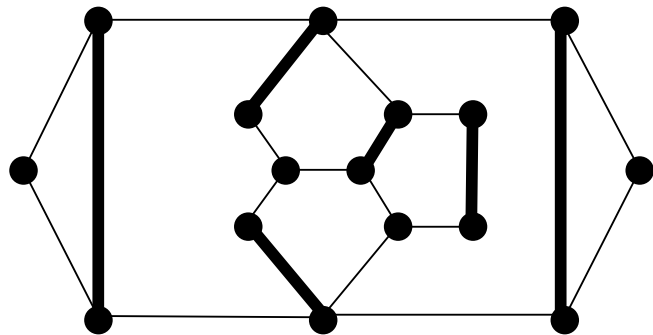
2. Find a minimum-weight perfect matching and a maximum-weight vertex under-cover for the bipartite graphs with weight matrices given above. (*Assigned: d*)

3. In the graphs below, perform the blossom algorithm with the given matchings to either find a perfect matching or a set A such that $oc(G - A) > |A|$.



a.

(Use a tree rooted at x for this problem.)



b. (*Assigned!*)

4. Let X and Y be sets of size n where every element $x \in X$ has a total ranking of the elements of Y , and every $y \in Y$ has a total ranking of the elements of X . Let M be a stable matching found by the Gale-Shapely proposal algorithm with the elements of X proposing, and let M' be any stable matching.

- a. (*Assigned!*) Prove that for every $x \in X$, x prefers its match in M to its match in M' .
- b. Prove that for every $y \in Y$, y prefers its match in M' to its match in M .

5. Find stable matchings for the rankings below.

$a : x > y > z > w$	$w : a > b > d > c$	$h : p > q > r > s > t$	$p : i > j > \ell > k > h$
$b : x > y > w > z$	$x : c > d > b > a$	$i : p > s > q > t > r$	$q : j > i > h > k > \ell$
$c : w > y > z > x$	$y : d > c > b > a$	$j : r > s > t > q > p$	$r : i > \ell > k > j > h$
$d : x > w > z > y$	$z : a > d > c > b$	$k : p > r > t > s > q$	$s : k > j > \ell > h > i$
	a.	$\ell : q > r > p > t > s$	b.