MATH 482, Spring 2013 - Homework 6

Due Wednesday 12/04

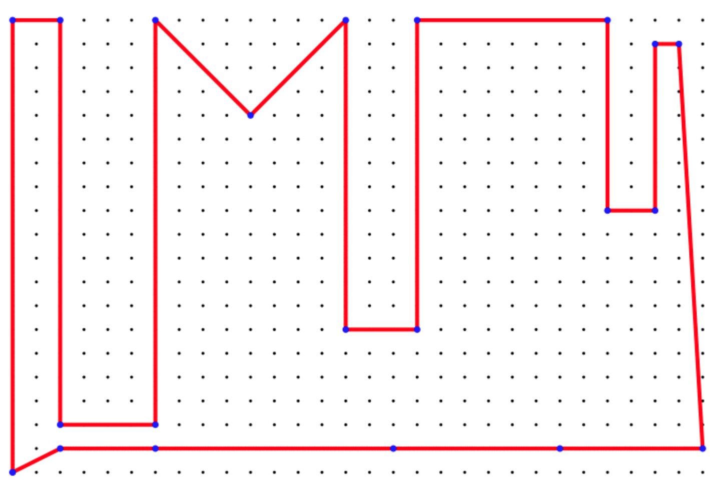
Assigned: 1, 4, 5, 7.

1. (*Assigned!*) Prove that Prim's Algorithm produces a minimum-weight spanning tree. (*Hint:* Use a proof format similar to the proof of Kruskal's Algorithm.)

2. Consider the weighted graph with weighted adjacency matrix below (blank positions represent non-edges). Find a minimum-weight spanning tree.

$$\begin{bmatrix} & 3 & 4 & 2 & 6 & \\ 3 & & 5 & & 4 & 7 \\ 4 & 5 & & 3 & & 2 \\ 2 & & 3 & & 4 & 3 \\ 6 & 4 & & 4 & & 5 \\ & 7 & 2 & 3 & 5 & \end{bmatrix}$$

3. Consider the points below on the integer grid and the given non-optimal tour. Create an improved tour by following at least one iteration of the δ-path algorithm.



4. (*Assigned!*) Consider the weighted graph with weighted adjacency matrix below (blank positions represent non-edges). Compute all six 1-tree lower bounds on a minimum-weight traveling salesman tour.

$$\begin{bmatrix} & 5 & 2 & 9 & & \\ 5 & & 3 & & 4 & 9 \\ 2 & 3 & & 2 & & 7 \\ 9 & & 2 & & 4 & \\ & 4 & & 4 & & 8 \\ & 9 & 7 & & 8 & \end{bmatrix}$$

5. (*Assigned!*) Let G be a graph and let

$$\mathcal{I} = \{J \subseteq E(G) : \text{each component of the subgraph with edges in } J \text{ contains at most one cycle}\}.$$

Prove that $(E(G), \mathcal{I})$ is a matroid.

6.
a. Let $\mathcal{M} = (S, \mathcal{I})$ be a matroid. Prove that the rank function is *submodular*: that for all $A, B \subseteq S$ we have

$$r(A) + r(B) \geq r(A \cup B) + r(A \cap B).$$

b. Prove that a function r on the subsets of S is the rank function of a matroid over S if and only if it is submodular and also satisfies $r(\varnothing) = 0$ and $r(A \cup \{x\}) \in \{r(A), r(A) + 1\}$ for all $A \subset S$ and $x \in S \setminus A$. (*Hint:* Construct a matroid $(S, \mathcal{I})$ where $\mathcal{I}$ is defined using the rank function r.)

7. (*Assigned!*) Present an implementation of the greedy algorithm that performs at most $|S|$ matroid queries of the type "Is A in $\mathcal{I}$?"