Fall 2013 Math 566 Implementation Assignment 0 Shortest Paths

Problem: Shortest Paths in Digraphs

Given a directed graph G with edge weights w and a pair of vertices s, t, return a shortest st-path and output its weight. Also, output a solution to the dual problem. Your implementation should use either Dijkstras algorithm, the Primal-Dual algorithm for linear programming, or a sufficiently similar algorithm. Describe the details of your implementation, and be sure your report answers the questions below. Finally, report the output of your implementation on the given problem instances.

Questions To Answer.

Q1. Formally define the problem and the dual problem using linear programming.
Q2. Provide a combinatorial interpretation of the dual problem.
Q3. Prove that strong duality holds for the combinatorial primal and dual problems by showing that the linear primal and dual problems have integral optimal solutions.
Q4. What language, libraries, and environments did you use?
Q5. What challenges did you encounter during your implementation?
Q6. What online/library resources did you use?

Problem Instances.

I1 & I2. Consider the digraphs with the given weighted adjacency matrices A_1 and A_2, where the first vertex is s and the last vertex is t.

$$A_1 = \begin{bmatrix} \infty & 3 & 5 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & 7 & 5 & \infty & \infty \\ \infty & \infty & \infty & 2 & \infty & 3 & \infty \\ \infty & \infty & \infty & \infty & 6 & 2 & \infty \\ \infty & \infty & \infty & \infty & \infty & 3 & 1 \\ \infty & \infty & \infty & \infty & \infty & \infty & 5 \\ \infty & \infty & \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

$$A_2 = \begin{pmatrix}
\infty & 2 & 5 & \infty & 8 & 9 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & 7 & 6 & \infty & \infty & 4\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \\
\infty & \infty & \infty & 8 & 7 & \infty & 6 & 6 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & 2 & 9 & 8 & 1 & 2 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & 5 & 1 & \infty & 5 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & 1 & 1 & 8 & \infty & 4 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & 2 & 6 & \infty & 10 & 6 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 6 & 3 & 7 & 5 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 9 & 5 & 3 & 4 & 1 & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 3 & 8 & 9 & 10 & 1 & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 1 & 9 & \infty & \infty & 9 & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 10 & 5 & 10 & 2 & 5 & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 7 & 9 & 1 & 8 & 10 & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 3 & \infty & 2 & 7 & 6 & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 9 & 4 & \infty & 8 & 9 \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 2 & 6 & 2 & 8 \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 7 & 5 & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 3 & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty
\end{pmatrix}$$

I3. See the file `reach1.txt` on the course web page.
I4. See the file `reach2.txt` on the course web page. You do not need to produce a dual solution for this instance.