Fall 2013 Math 566 Implementation Assignment 4

Problem: Branch-and-Bound Techniques for ILP

In this report, you will use a linear programming solver to assist you in evaluating some integer programs using Branch-and-Bound. It is not expected that you build a full algorithm to automate a branch-and-bound algorithm, but you must follow such an algorithm. You are not to use an integer programming solver.

With each integer program below, you will describe the solution and the branch-and-bound tree, including: In what order did you expand the nodes? What variable branches were used? How did you select which node to branch on? How did you select which variable to branch on? For every leaf: why did it terminate?

Questions to Answer in Your Report

Q1. What linear programming solver did you use?
Q2. How did you store your branch-and-bound tree?
Q3. How did you encode the branching constraints?
Q4. What was the most difficult part of this assignment?

Problem Instances.

I1.

$$
\begin{array}{rrcrcrcrl}
\min & 3x_1 & + & x_2 & + & 2x_3 & & & \\
\text{subject to} & x_1 & + & x_2 & + & 2x_3 & \geq & 6 & \\
 & & & 12x_2 & - & x_3 & \geq & 5 & \\
 & 5x_1 & - & x_2 & & & \geq & 0 & \\
 & x_1, & & x_2, & & x_3 & \geq & 0 & \\
 & x_1, & & x_2, & & x_3 & & \text{integer} &
\end{array}
$$

I2.

$$
\begin{array}{rccccccccccl}
\min & & & & & & & & & x_5 & & \\
\text{subject to} & 2x_1 & + & 2x_2 & + & 2x_3 & + & 2x_4 & + & x_5 & = & 5 \\
 & x_1, & & x_2, & & x_3, & & x_4, & & x_5 & \in & \{0,1\}
\end{array}
$$

I3.

$$
\begin{array}{rccccccccccl}
\min & 25x_1 & + & 16x_2 & + & 9x_3 & + & 4x_4 & + & x_5 & & \\
\text{subject to} & x_1 & + & x_2 & + & x_3 & & & & & \geq & 1 \\
 & -x_1 & - & x_2 & & & + & x_4 & & & \geq & -1 \\
 & -x_1 & & & & & - & x_4 & + & x_5 & \geq & 0 \\
 & & & x_2 & - & x_3 & & & + & x_5 & \geq & 0 \\
 & & & & & x_3 & - & x_4 & - & x_5 & \geq & -1 \\
 & x_1 & & & & & + & x_4 & - & x_5 & \geq & 0 \\
 & x_1, & & x_2, & & x_3, & & x_4, & & x_5 & \in & \{0,1\}
\end{array}
$$

I4.

$$
\begin{array}{rccccccccccl}
\min & x_1 & + & x_2 & + & x_3 & + & x_4 & + & x_5 & + & x_6 & \\
\text{subject to} & x_1 & + & x_2 & & & & & & & & & \geq \frac{5}{3} \\
 & x_1 & & & + & x_3 & & & & & & & \geq \frac{3}{4} \\
 & & & x_2 & & & + & x_4 & & & + & x_6 & \geq \frac{3}{2} \\
 & & & & & & & & & x_5 & + & x_6 & \geq \frac{1}{2} \\
 & & & & & x_3 & + & x_4 & + & x_5 & & & \geq 1 \\
 & x_1, & & x_2, & & x_3, & & x_4, & & x_5, & & x_6 & \geq 0, \text{integer}
\end{array}
$$