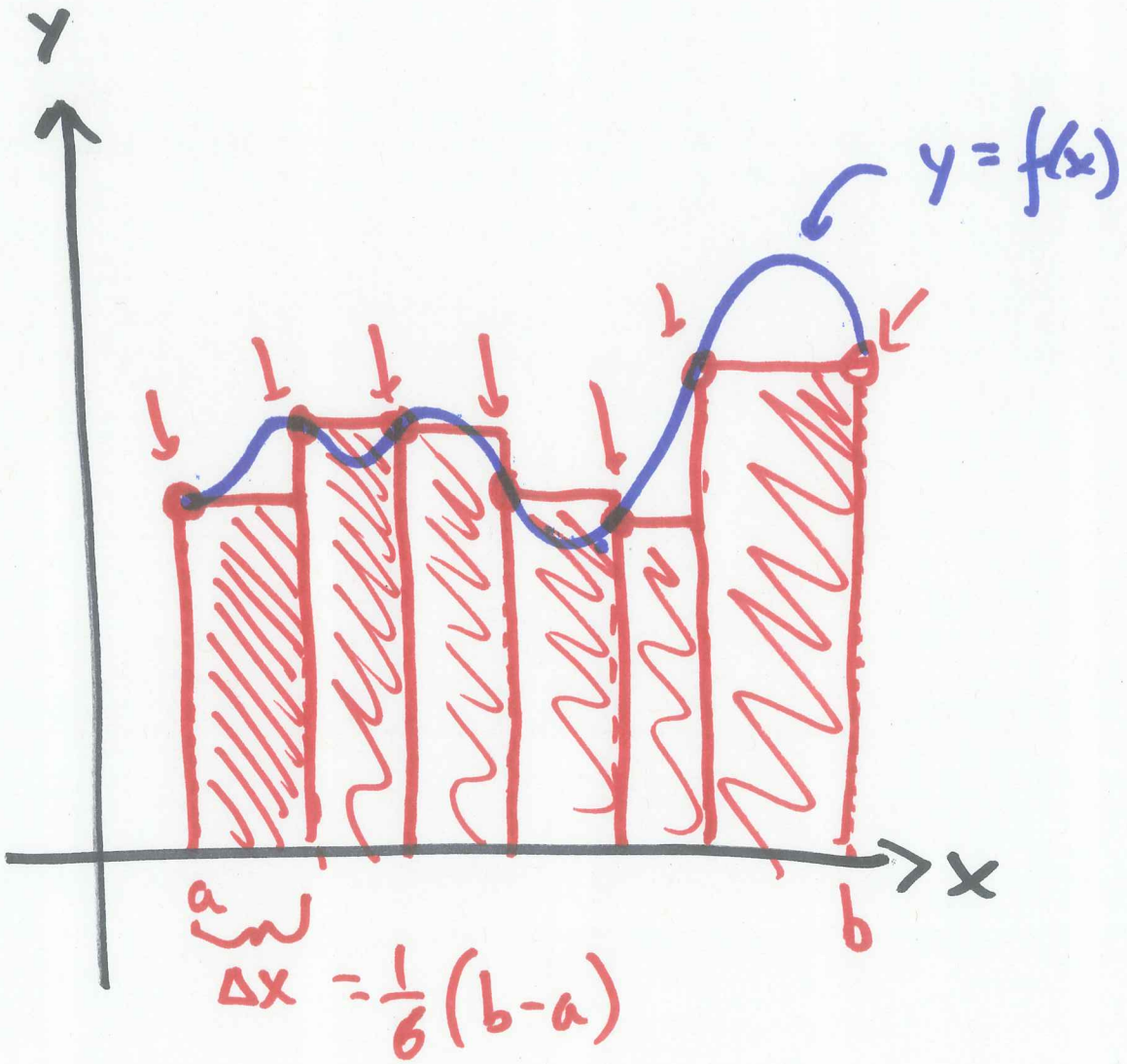
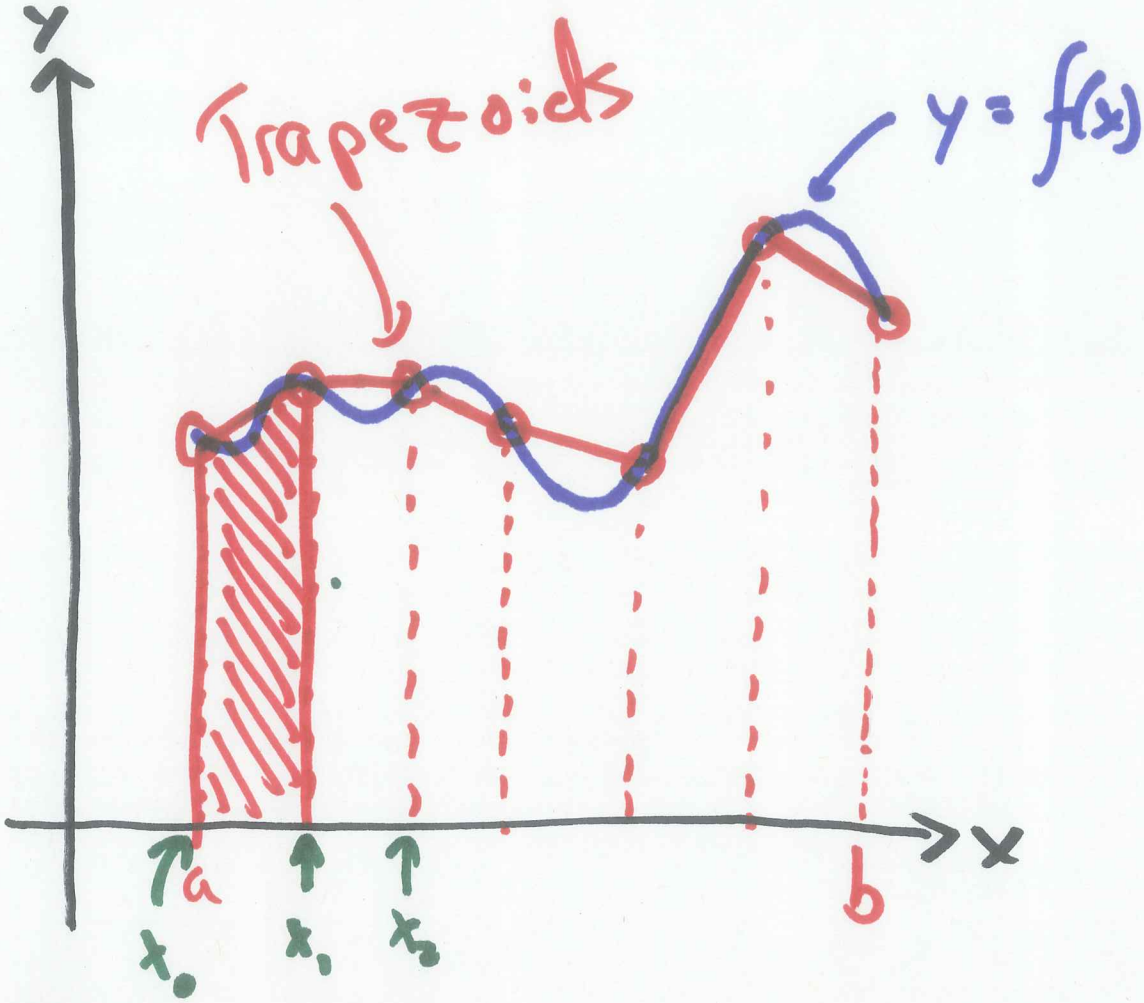
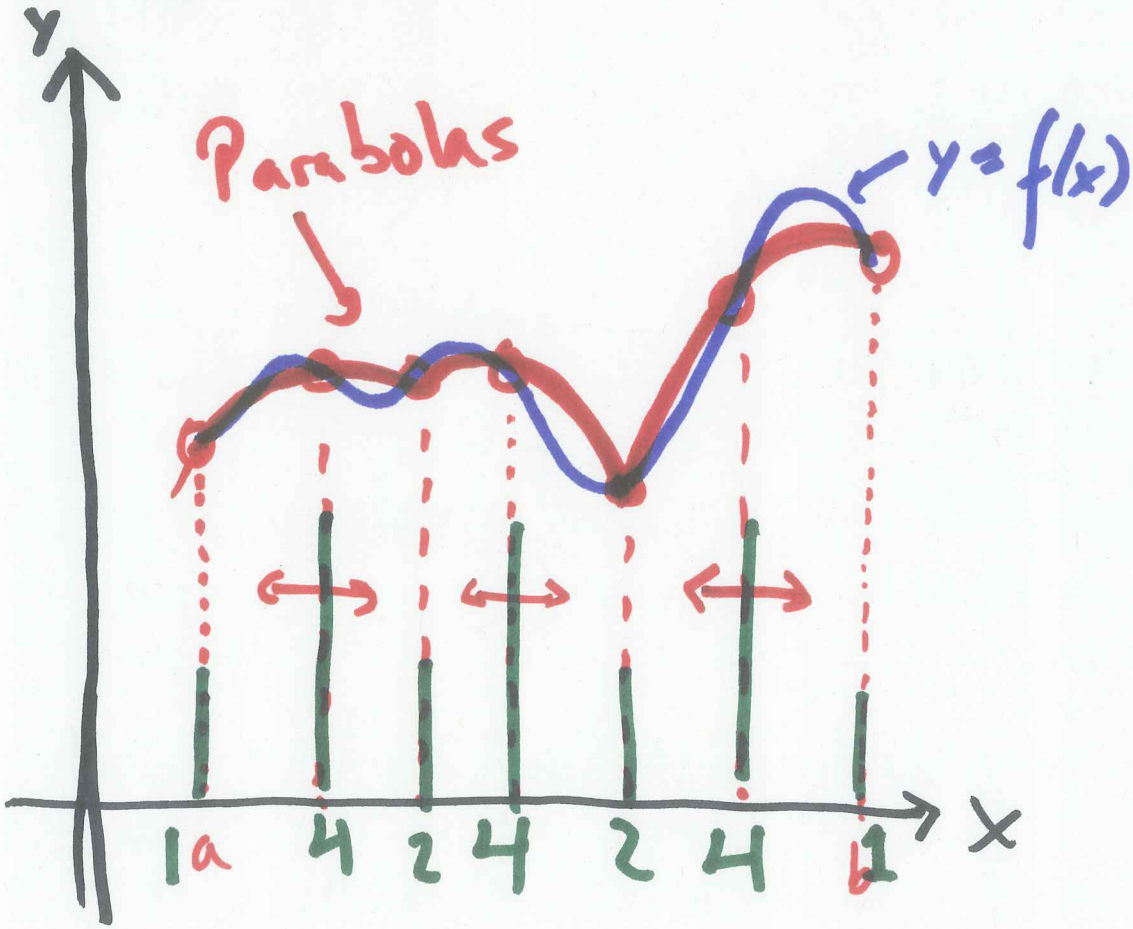
8.6 Numeric Integration

What if an integral is too difficult to solve exactly? Approximate!

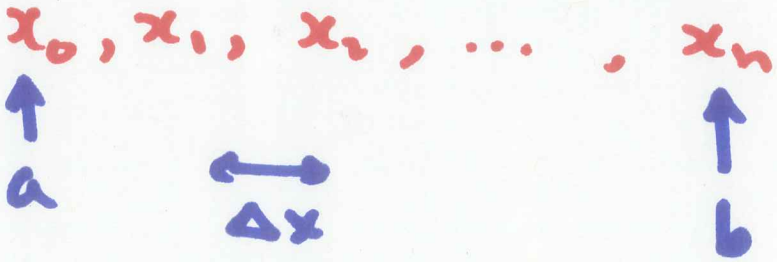



y
Trapezoids
y = f(x)
x
a
b
x_0
x_1
x_2


y
Parabolas
y = f(x)
x
1a
4
2
4
2
4
1
b

Approximations

1. Determine interval $[a,b]$
2. Determine # of steps n
3. Determine step size $\Delta x = \frac{b-a}{n}$
4. Determine steps



$$x_i = a + i \cdot \Delta x$$

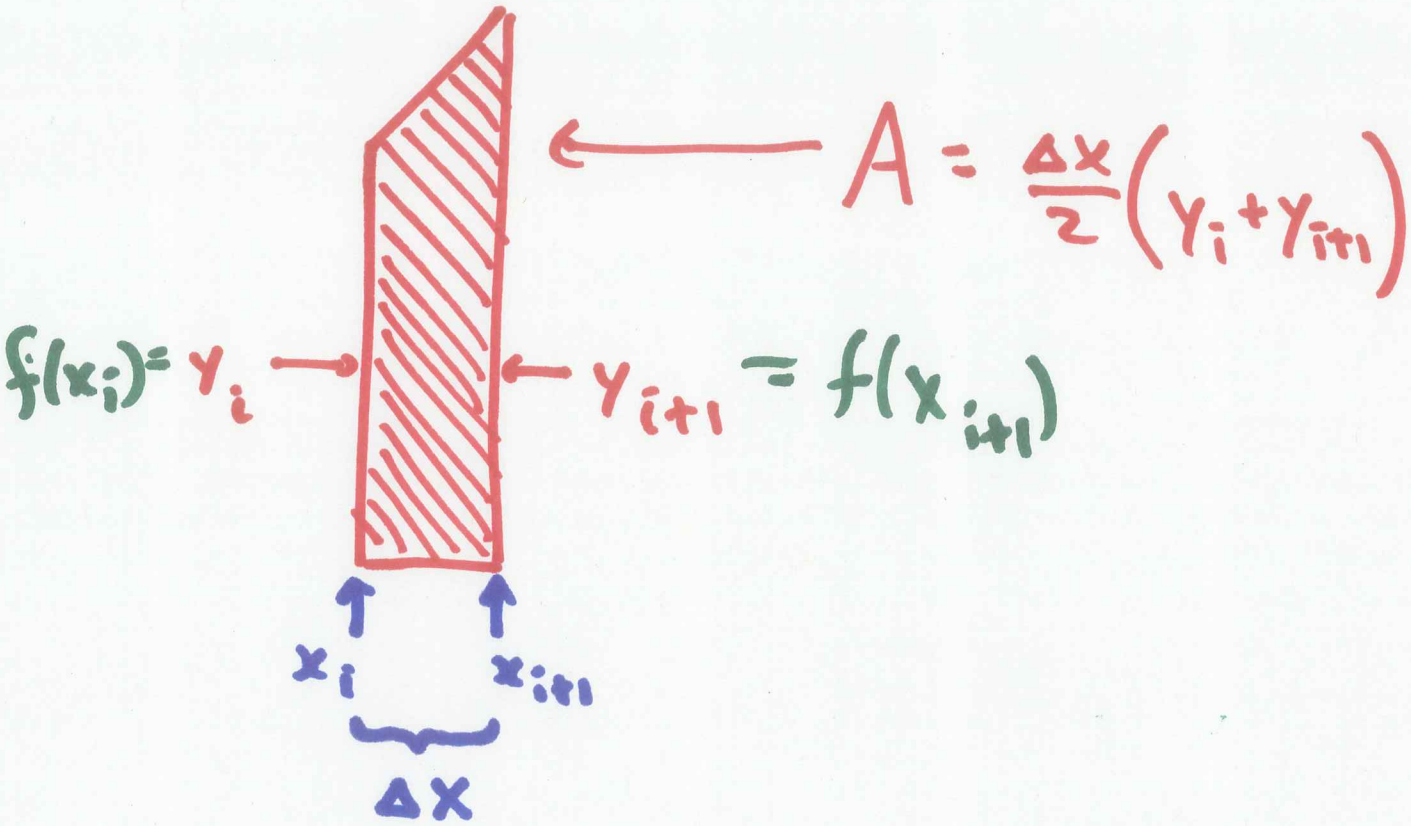
$$= a + i\left(\frac{b-a}{n}\right)$$

5. Determine f-values $f(x_i) = y_i$

Approximation Table

i	x_i	$f(x_i)$

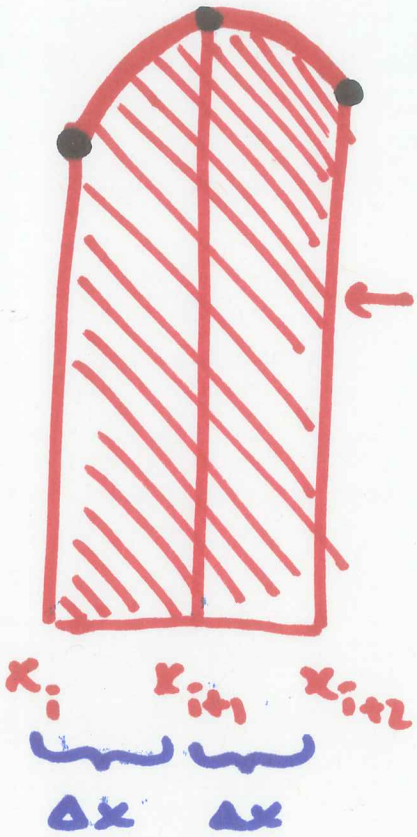
Trapezoid Rule



$$T = \frac{\Delta x}{2}\left(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n\right)$$

$$|E_T| \leq \frac{M(b-a)^3}{12\,n^2}$$ ← Error Bound

Simpson's Rule (Parabolas)



$$A = \frac{\Delta x}{3}\left(y_i + 4y_{i+1} + y_{i+2}\right)$$

$$S = \frac{\Delta x}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right)$$

$$|E_S| \leq \frac{M(b-a)^5}{180\,n^4}$$ ← Error Bound

Error Analysis

$f, a, b,$ as before.

n: # of intervals

f'': double derivative of f

M: upper bound on $|f''|$ over $[a,b]$

E_T: $\int_a^b f(x)dx - T$ $\quad |E_T| \leq \frac{M(b-a)^3}{12n^2}$

E_S: $\int_a^b f(x)dx - S$ $\quad |E_S| \leq \frac{M(b-a)^5}{180n^4}$

Ex: $f(x) = \sqrt{\sin^2 x^2 + \cos^2 x^3}$