$$\int e^{x}\sqrt{e^{2x}+1}\,dx$$

$$\int \underline{\sin^{3}\theta\ \cos^{2}\theta\,d\theta}$$

$$\int \sin^{3}\theta\,\cos^{3}\theta\,d\theta$$

$$\swarrow$$

$$\int \sin\theta(1-\cos^{2}\theta)\cos^{3}\theta\,d\theta$$

$$\mu=\cos\theta$$

$$du=-\sin\theta\,d\theta$$

$$\int \frac{dx}{(x+1)(x-1)}$$

$$\begin{tikzcd}
& \int \sin^2\theta \cos^2\theta \, d\theta \arrow[dl] \arrow[dr] & \\
\frac{1-\cos 2\theta}{2} & & \frac{1+\cos 2\theta}{2}
\end{tikzcd}$$

$$= \frac{1}{4}\int (1-\cos 2\theta)(1+\cos 2\theta)\, d\theta$$

$$= \frac{1}{4}\int \left(\underline{1-\cos^2 2\theta}\right) d\theta$$

$$\downarrow$$

$$= \frac{1}{4}\int \sin^2 2\theta \, d\theta$$

$$\downarrow$$

HAF again!

$$\downarrow$$

$$\frac{1}{4}\int \left(\frac{1-\cos 4\theta}{2}\right) d\theta$$

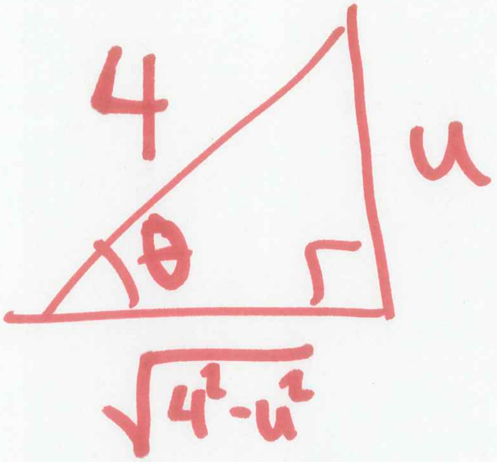
$$\int \frac{\boxed{x}}{\sqrt{16-x^4}}\,\boxed{dx}$$

$u=x^2$

$du=2x\,dx$

$$\frac{1}{2}\int \frac{du}{\sqrt{4^2-u^2}}$$

$-4<u<4$



$u=4\sin\theta$

$du=4\cos\theta\,d\theta$

$\sqrt{4^2-u^2}=4\cos\theta$

$-\frac{\pi}{2}<\theta<\frac{\pi}{2}$

$$\frac{1}{2}\int \frac{\cancel{4\cos\theta}\,d\theta}{\cancel{4\cos\theta}}$$

$$=\frac{1}{2}\theta+C$$

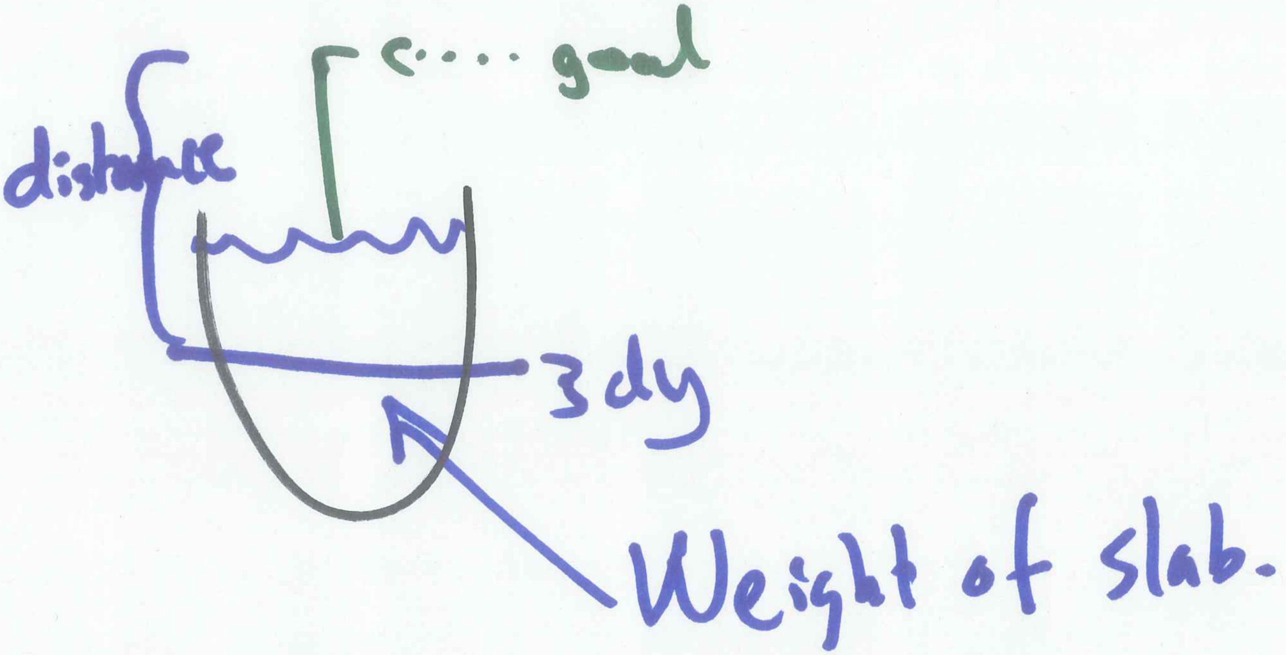
$$=\frac{1}{2}\arcsin\left(\frac{u}{4}\right)+C$$

$$=\frac{1}{2}\arcsin\left(\frac{x^2}{4}\right)+C$$

Ex: $$\int \frac{x}{\sqrt{4x^2-1}}\,dx$$

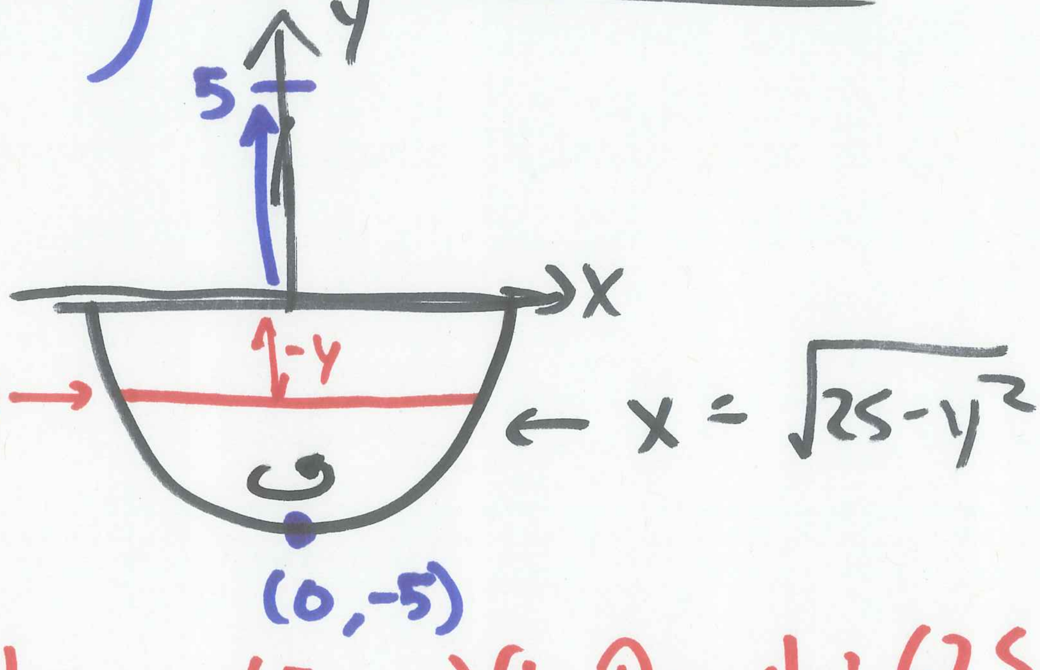
3 ways:

1. $u=2x$
2. Use Hypotenuse $2x$ (DANGER)
3. $$\int \frac{x}{2\sqrt{x^2-\frac{1}{4}}}\,dx$$



$$W = \int (\text{Distance})\cdot(\text{Weight})$$

Ex:



Distance: $(5-y)\,ft$ Density: $62.5\ \frac{lbs}{ft^3}$

$$\text{Weight}: 62.5\cdot \underbrace{\pi\, \overset{ft}{\boxed{r}}^{2}}\ \boxed{dy} \leftarrow ft$$

$$= \underset{\uparrow\ lbs/ft^3}{62.5}\ \underbrace{\pi(25-y^2)dy}_{ft^3} \leftarrow lbs$$

$$W = \int \text{Distance} \times \text{Weight}$$

$$= \int_{-5}^{0} \underbrace{(5-y)}_{ft} \; \underbrace{62.5\pi\left(25-y^2\right)dy}_{lbs}$$

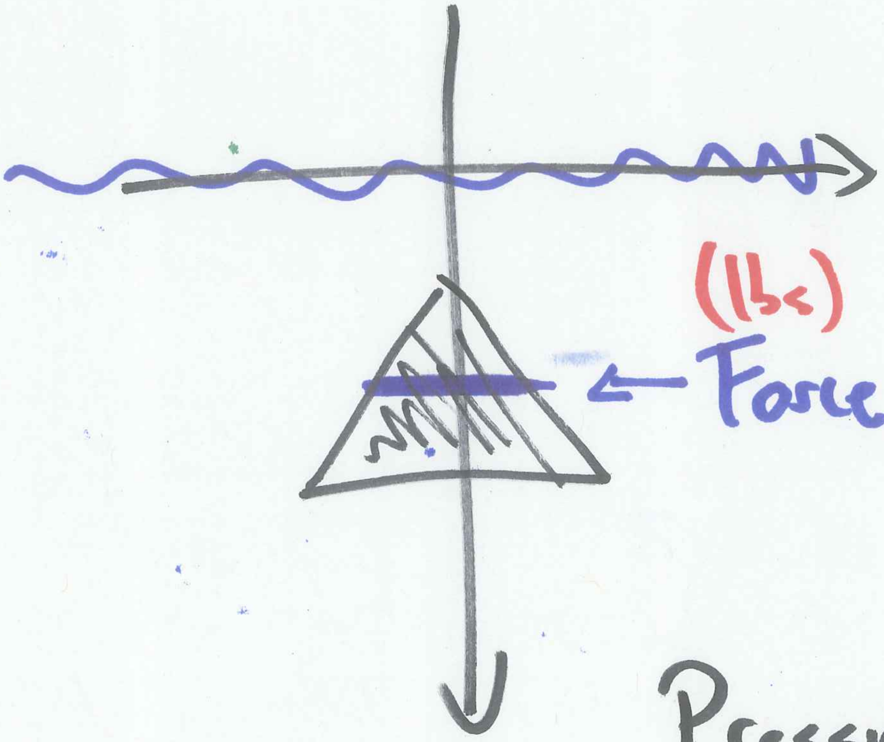
etc.

Bounds: Where is liquid?

TIP: Unit checks!

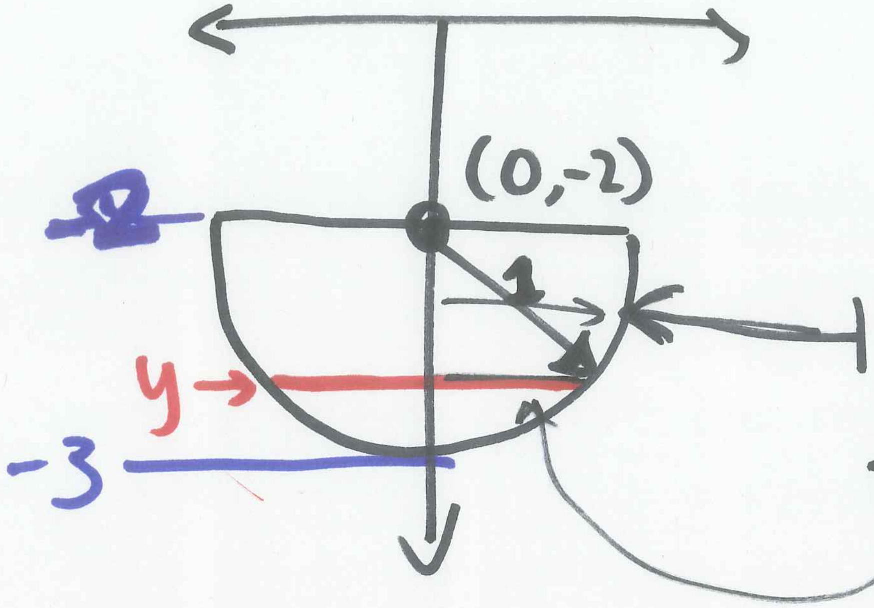
Pumping : Container w/ liquid
Move the liquid to a place.

Fluid Force: Plate in a fluid
Measure the force by the pressure



$$\overset{(\text{lbs})}{\text{Force}} = \overset{(\text{lbs}/\text{ft}^2)}{\text{Pressure}} \times \overset{(\text{ft}^2)}{\text{Area}}$$

$$\underset{(\text{lbs}/\text{ft}^2)}{\text{Pressure}} = \overset{(\text{lbs}/\text{ft}^3)}{\text{Density}} \times \underset{(\text{ft})}{\text{Depth}}$$



$$x^2 + (y+2)^2 = 1$$

$$x = \sqrt{1-(y+2)^2}$$

Depth: $-y = 0 - y$

Density: $62.5 \frac{lbs}{ft^3}$

Area: $\underbrace{\text{Width}}_{2x = 2\sqrt{1-(y+2)^2}} \times \underbrace{\text{Height}}_{dy}$

$$\text{Force} = \int \text{Pressure} \times \text{Area}$$

$$= \int_{-3}^{-2} \overbrace{62.5\cdot(-y)}\cdot\overbrace{2\sqrt{1-(y+2)^2}\,dy}$$