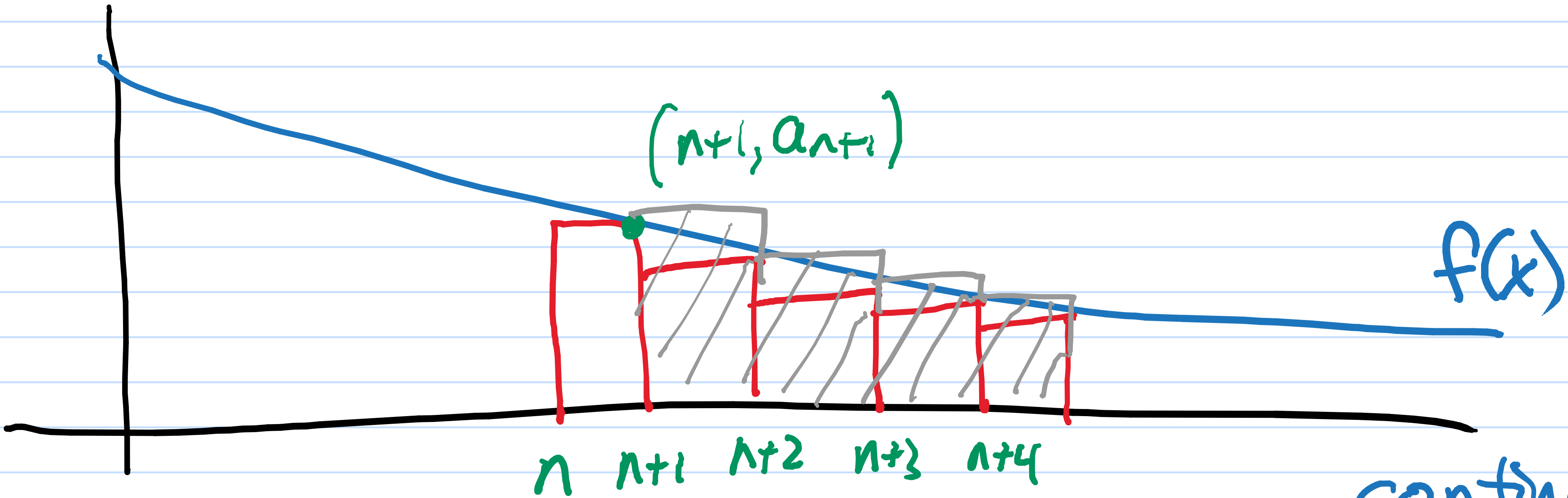

continuous
positive
decreasing

$$\sum_{k=n+1}^{\infty} a_k \leq \int_{n}^{\infty} f(x)\,dx$$

$$\sum_{k=n+1}^{\infty} a_k \geq \int_{n+1}^{\infty} f(x)\,dx$$

Why Taylor Series?

$$x^2 f''(x) + \sin x \, f'(x) - \frac{1}{x} f(x) = e^{-x^2}$$

Can't solve that

Let $f(x) = \sum_{n=0}^{\infty} a_n x^n$

Maybe figure out $P_3 = \sum_{n=0}^{3} a_n x^n$

Power Series

$$\sum_{n=0}^{\infty} a_n (x-a)^n$$

radius of convergence

interval of convergence

- multiply power series
- substitute $f(x)$ for x in a power series
- differentiate power series
- integrate power series

$$a-R < x < a+R \qquad R = ROC$$

Taylor / Maclaurin series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n \qquad a-R<x<a+R$$

$$e^x \longrightarrow e^{-x^2/2}$$

$\sin x$

$\cos x$

$\frac{1}{1-x}$

$$P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

Let $R_n(x) = f(x) - P_n(x)$

"remainder"

There exists a c between x and a such that

$$R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$$

Let $M = \max\limits_{\substack{c \text{ betw.} \\ x \,\&\, a}} \{|f^{(n+1)}(c)|\}$

$$|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1}$$

$$(1+x)^m = 1 + \sum_{k=1}^{\infty} \binom{m}{k} x^k$$

$$\binom{m}{k} = \frac{m(m-1)\cdots(m-k+1)}{k!} \qquad k \geq 1$$

for $-1 < x < 1$

Ch. 10 Practice #3

Does $a_n = \dfrac{1-2^n}{2^n}$ converge or diverge?

(Sequence)

$$\lim_{n\to\infty} \frac{1-2^n}{2^n} = \lim_{n\to\infty}\left(\frac{1}{2^n} - 1\right)$$

We know $\displaystyle\lim_{x\to\infty}\left(\frac{1}{2^x} - 1\right) = -1$

Therefore, $\displaystyle\lim_{n\to\infty}\left(\frac{1}{2^n} - 1\right) = -1$

__Problem__ Does $\sum_{n=0}^{\infty} \frac{1-2^n}{2^n}$ converge or diverge?

Diverges by n^{th} term test

$$\lim_{n\to\infty} \frac{1-2^n}{2^n} = -1 \neq 0$$

Example 10.8 #22

Find the Maclaurin series for

$$\frac{x^2}{x+1} = x^2 \frac{1}{1+x} = x^2 \frac{1}{1-(-x)}$$

$$\frac{1}{1-(-x)} = \sum_{n=0}^{\infty} (-x)^n = \sum_{n=0}^{\infty} (-1)^n x^n$$

$$\frac{x^2}{1-(-x)} = \sum_{n=0}^{\infty} (-1)^n x^{n+2}$$

$$= \sum_{k=2}^{\infty} (-1)^{k-2} x^k$$

Answer $= \sum_{k=2}^{\infty} (-1)^k x^k$

We could have said

$\frac{x^2}{1-(-x)}$ is the sum of a geometric series

$a = x^2$

$r = -x$