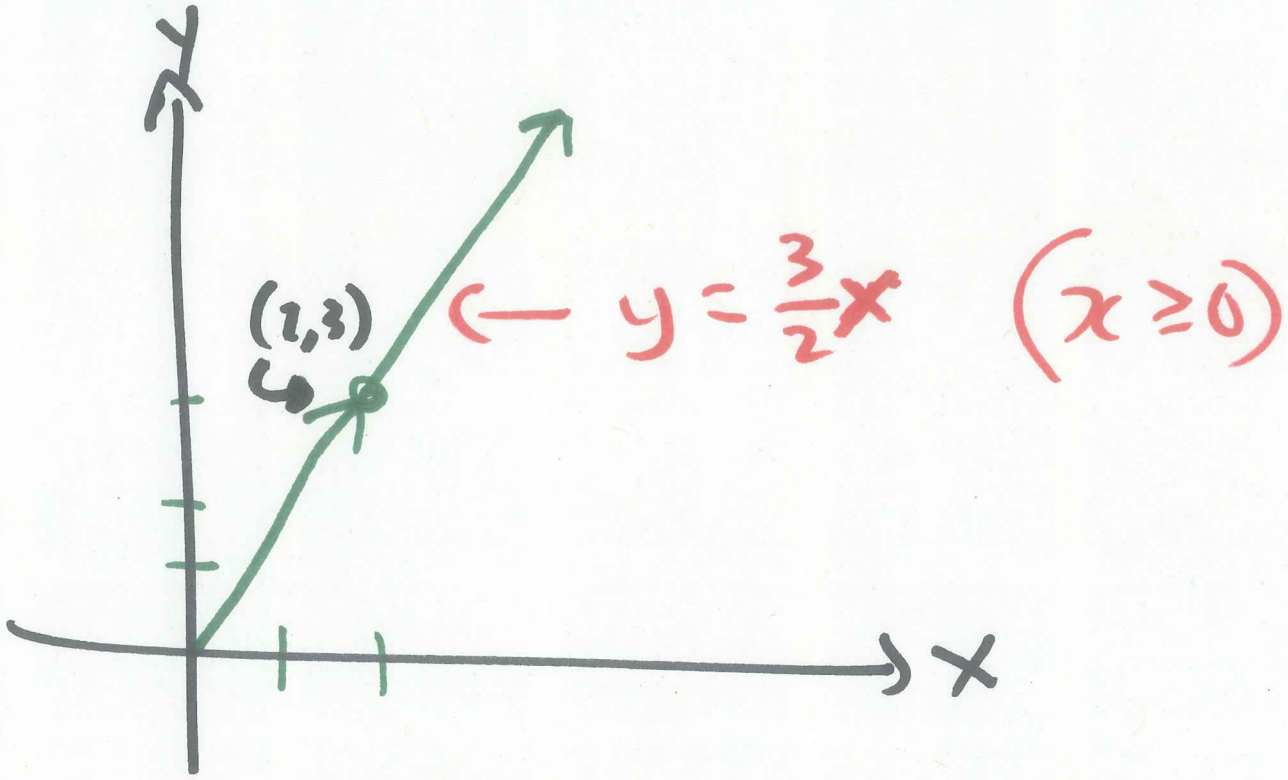
11.1: Parameterization of plane curves

A car starts at the origin of the xy-plane, then accelerates towards the point $(2, 3)$ at a rate of $\frac{1\text{m}}{\text{s}^2}$.
(2 m, 3 m)

What path does the car take?



$(x, y) = (f(t), g(t))$

$= \left(2 \cdot \frac{t^2}{2}, 3 \cdot \frac{t^2}{2}\right)$

NOT QUITE RIGHT!

Def: If x and y are given as functions

$$x = f(t), \quad y = g(t)$$

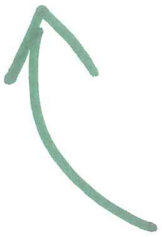
over an interval I of t-values, then the set of points $(x, y) = (f(t), g(t))$ defined by these equations is a parametric curve.

t: parameter.

Ex:

$$(x, y) = (\cos t, \sin t)$$

$$-\infty < t < \infty$$



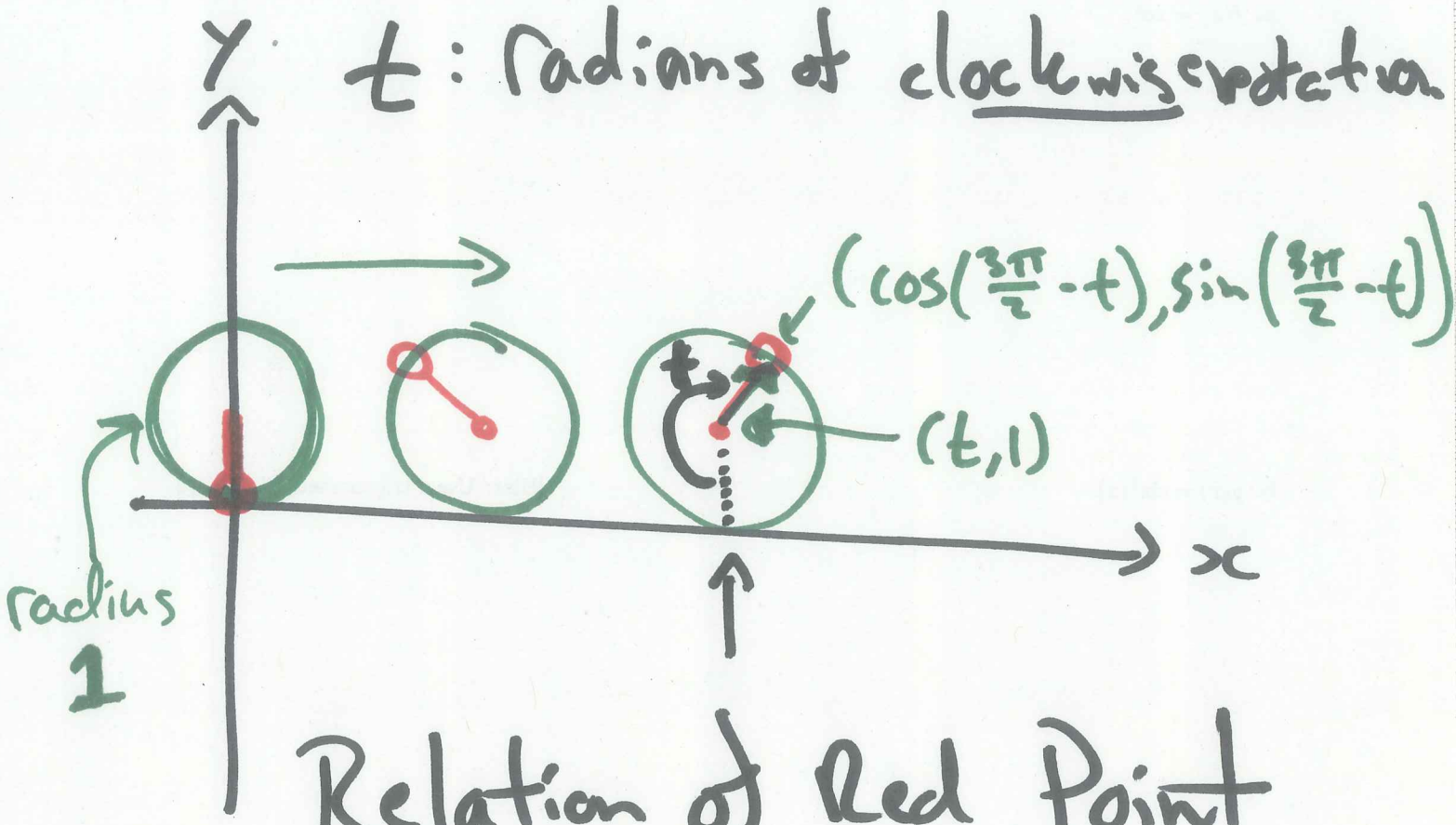
Unit Circle
(parametrized by angle t)

Implicit Def:

$$x^2 + y^2 = 1$$

$$\cos^2 t + \sin^2 t = 1$$

Ex: Cycloid



Relation of Red Point to Center:

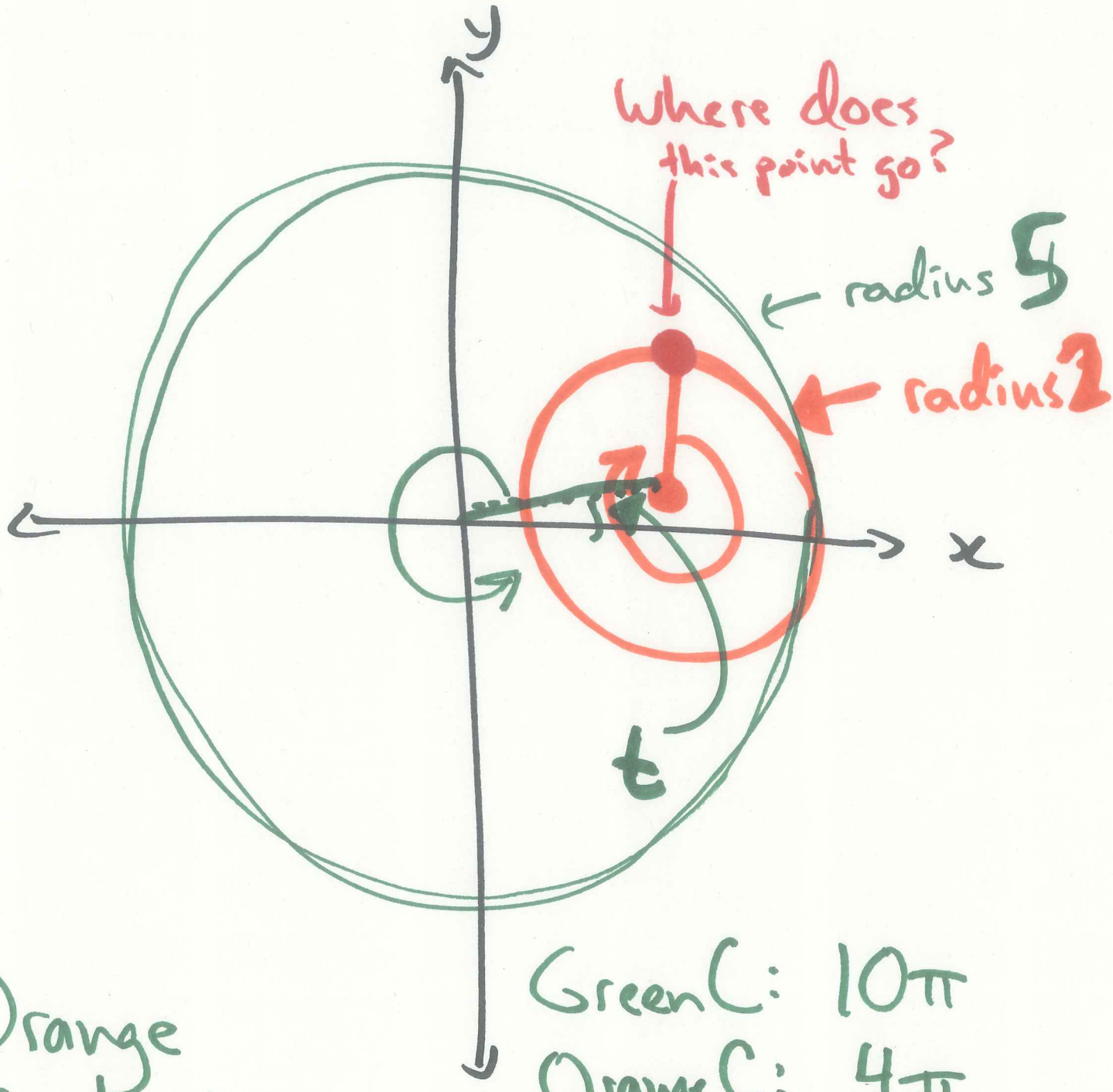
$$\left(\cos\left(\frac{3\pi}{2}-t\right), \sin\left(\frac{3\pi}{2}-t\right)\right)$$

Center: at $t=0$: $(0,1)$ → $(t,1)$

at $t=2\pi$: $(2\pi, 1)$

Param: $\left(t+\cos\left(\frac{3\pi}{2}-t\right), 1+\sin\left(\frac{3\pi}{2}-t\right)\right)$

Ex: Spirograph



Orange Centers:

$(3\cos t, 3\sin t)$

GreenC: 10π

OrangeC: 4π

$\frac{5}{2}$ rotations of O per rotation of G

Red Point w/ Lt orange center:

$$\left(2\cos\left(-\frac{5}{2}t\right), 2\sin\left(-\frac{5}{2}t\right)\right)$$

Opposite Dir.

Rotation Speed Diff

Param:

$$\left(3\cos t + 2\cos\left(-\frac{5}{2}t\right),\right.$$

$$\left.3\sin t + 2\sin\left(\frac{-5}{2}t\right)\right)$$