11.2 Calculus w/ Parametric Curves

$$x = x(t) = f(t)$$

$$y = y(t) = g(t)$$

Differentiable if f & g are differentiable at t.

dt = "change in t"

$$\frac{dx}{dt} = \frac{\text{"change in } x\text{"}}{\text{"change in } t\text{"}} = f'(t)$$

$$\frac{dy}{dt} = \frac{\text{"change in } y\text{"}}{\text{"change in } t\text{"}} = g'(t)$$

Parametric Formula for dy/dx

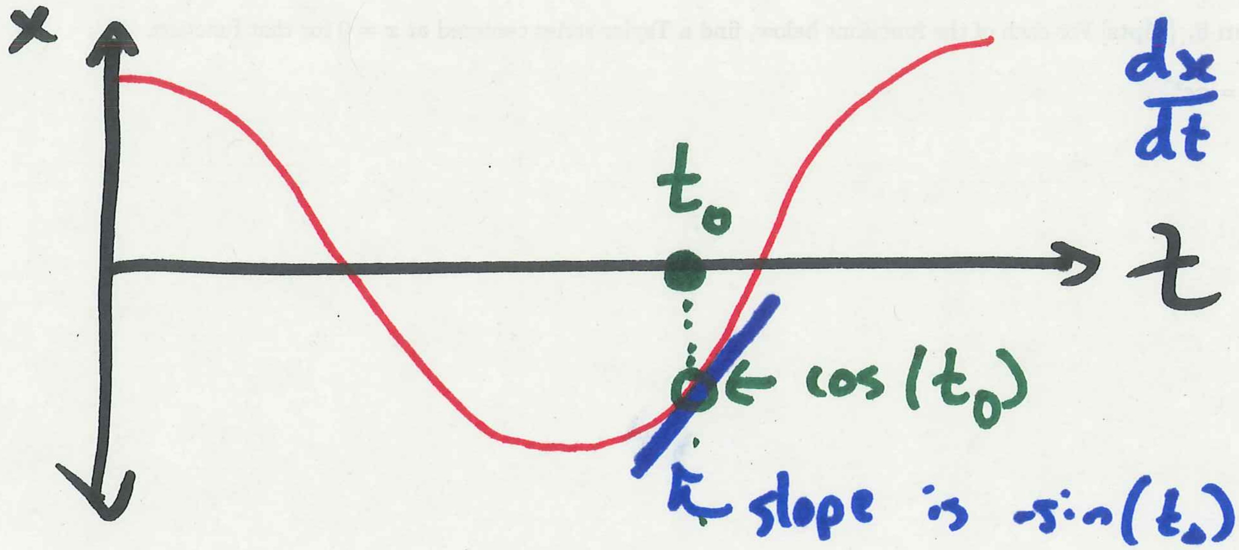
If $\frac{dx}{dt} \neq 0$, then

$$* \quad \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

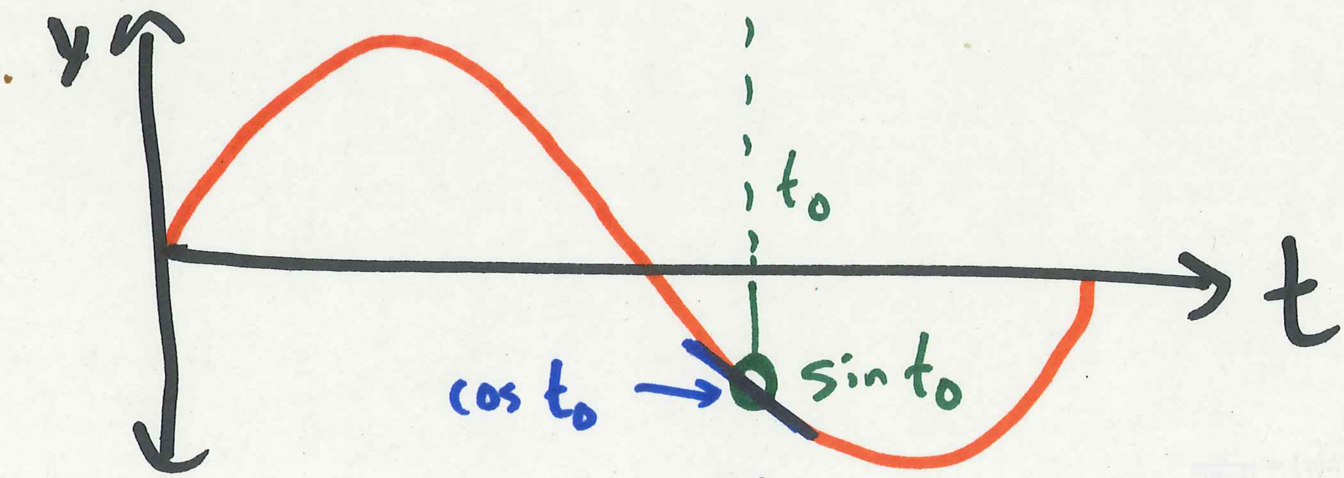
Chain Rule:

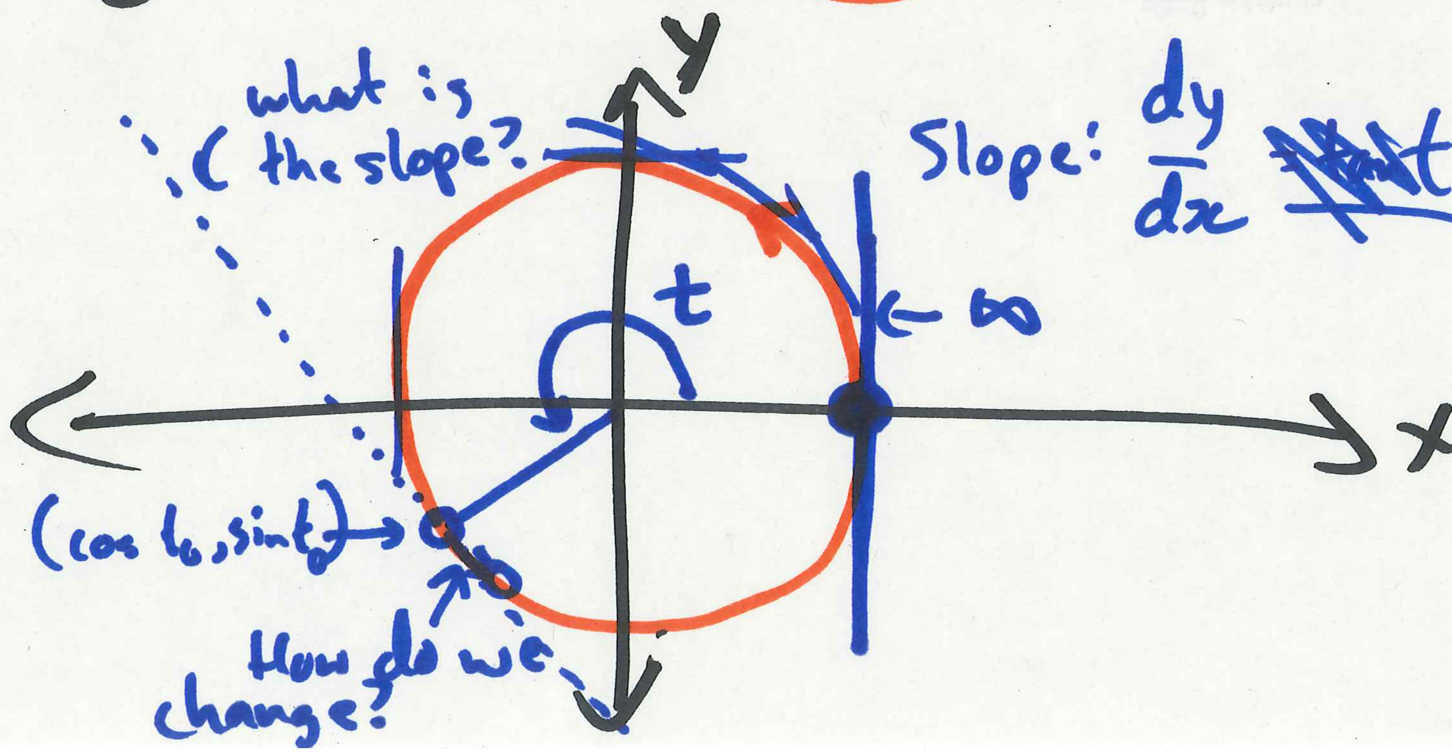
$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

Ex: $(x, y) = (\cos t, \sin t)$



$\frac{dx}{dt} = -\sin t$





Slope: $\frac{dy}{dx}$

$$(x,y) = (\cos t, \sin t)$$

$$\frac{dx}{dt} = -\sin t, \quad \frac{dy}{dt} = \cos t$$

$$\frac{dy}{dx} = \frac{\cos t}{-\sin t} = -\frac{\cos t}{\sin t} = \underline{-\cot t}$$

Ex: Let $(x,y) = (t^{-1}, \sin t)$.

Find the tangent line at $t = \pi$.

Point: $\left(\frac{1}{\pi}, 0\right)$ $\qquad (x,y) = (t^{-1}, \sin t)$

$$\frac{dx}{dt} = \frac{-1}{t^2} \xrightarrow[t=\pi]{\text{at}} \frac{-1}{\pi^2}$$

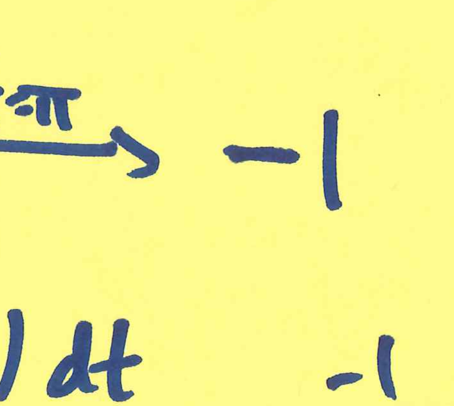
$$\frac{dy}{dt} = \cos t \xrightarrow{t=\pi} -1$$

At $t=\pi$: $\dfrac{dy/dt}{dx/dt} = \dfrac{-1}{-1/\pi^2} = \underline{\underline{\pi^2}}$

Slope.

Slope: π^2

Point: $\left(\frac{1}{\pi}, 0\right)$



$\approx (.3, 0)$

$y = \pi^2 x + -\pi$ $\quad (0, -\pi)$

Parametric Formula for $\frac{d^2y}{(dx)^2}$

If $\frac{dx}{dt} \neq 0$, $y' = \frac{dy}{dx}$, then

$$\frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt}$$

Ex: Find $\frac{d^2y}{dx^2}$ as a function of t if $(x,y) = (t^3 - t,\ t^2+1)$.

$$\frac{dx}{dt} = 3t^2 - 1, \quad \frac{dy}{dt} = 2t$$

$$y' = \frac{dy}{dx} = \frac{2t}{3t^2-1}$$

$$\frac{dy'}{dt} = \frac{(3t^2-1)(2) - (6t)(2t)}{(3t^2-1)^2}$$

$$= \frac{6t^2 - 2 - 12t^2}{(3t^2-1)^2}$$

$$= \frac{-6t^2-2}{(3t^2-1)^2}$$

$$\frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt}$$

$$= \frac{-6t^2-2}{(3t^2-1)^3}$$