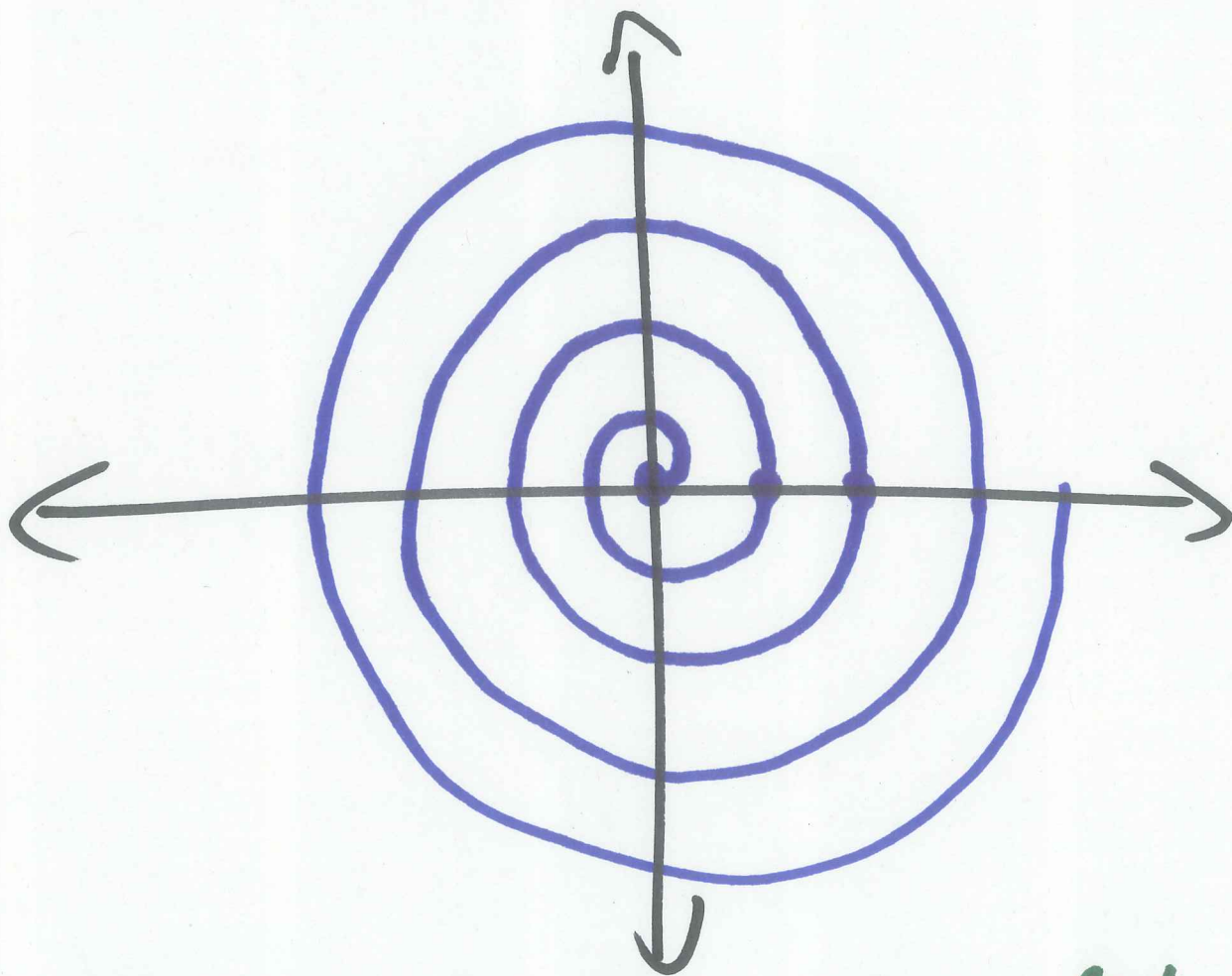


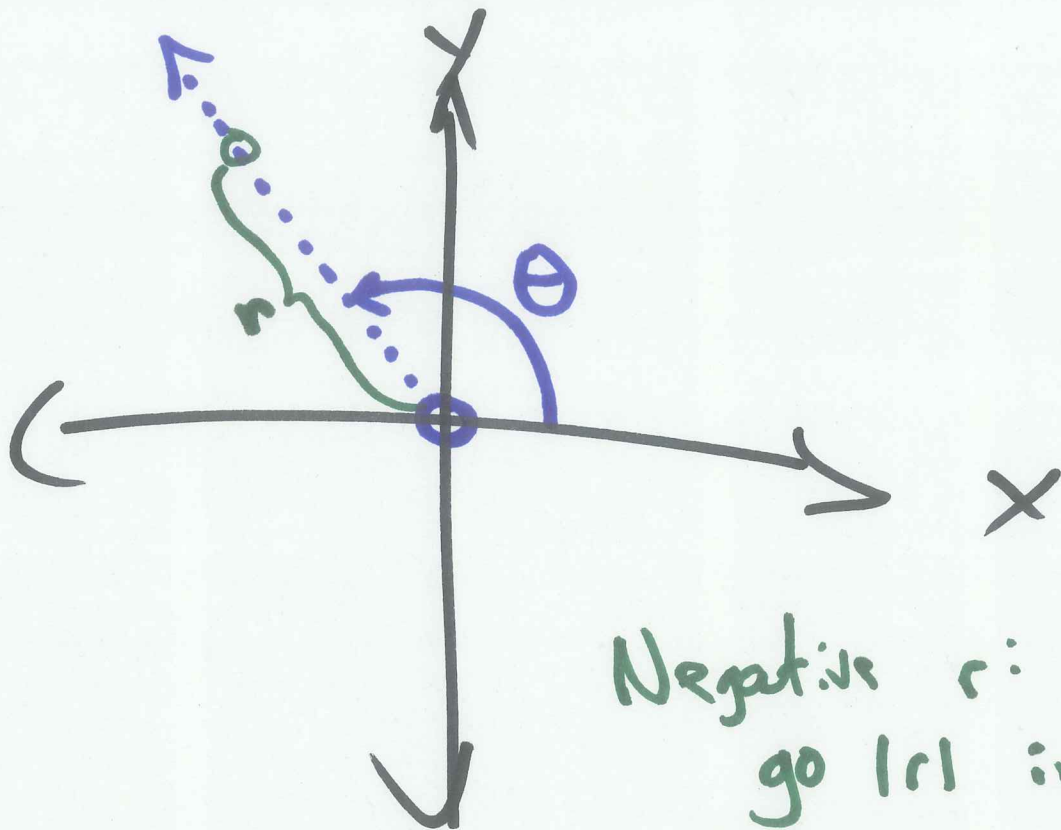
# Archimedean Spiral:

$$(x, y) = (t \cdot \cos(2\pi t), t \cdot \sin(2\pi t))$$
$$t \geq 0.$$

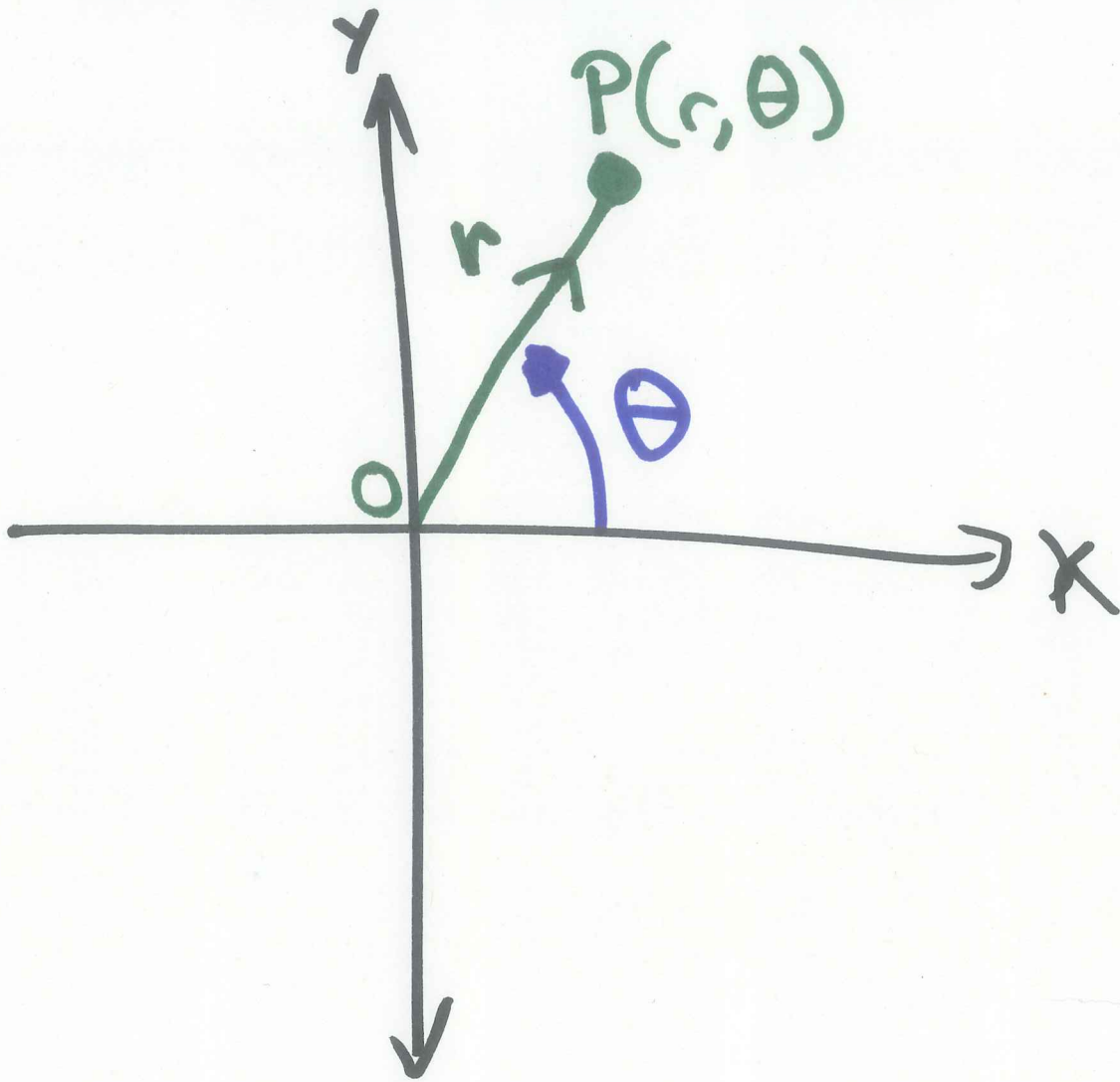


In Polar Coordinates:  $r = \theta / 2\pi$   
( $\theta \geq 0$ )

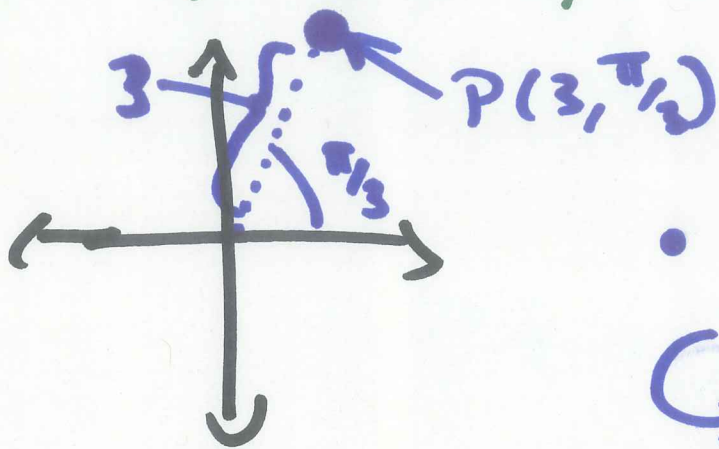
## 11.3: Polar Coordinates



# Polar Coordinates



Ex: Find all the polar coordinates of the point  $P(3, \pi/3)$ .



- $P(-3, 4\pi/3)$

↳  $P(-3, \frac{\pi}{3} + (2n+1)\pi)$   
(n integer)

↳  $P(3, \frac{\pi}{3} + 2n\pi)$

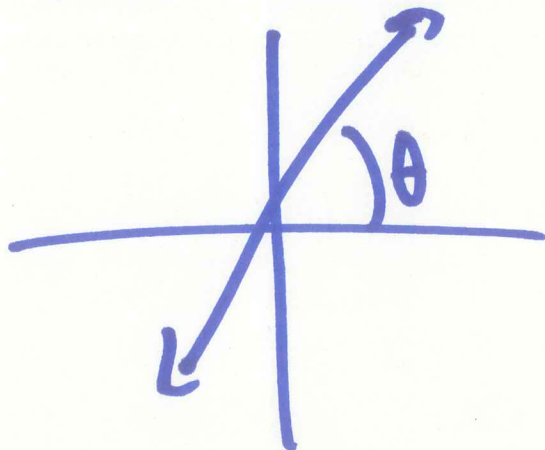
# Polar Equations

Ex:  $r = a.$

Circle of radius  $|a|.$

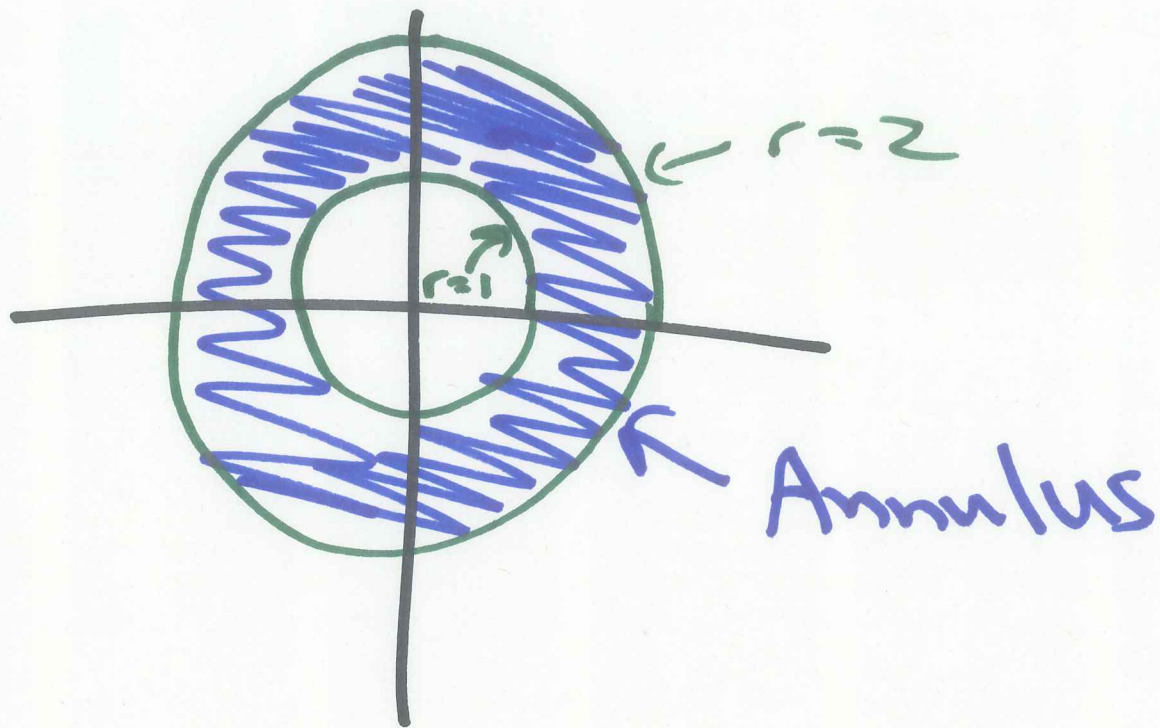
Ex:  $\theta = \theta_0.$

A line through the origin  
with slope  $\tan \theta.$

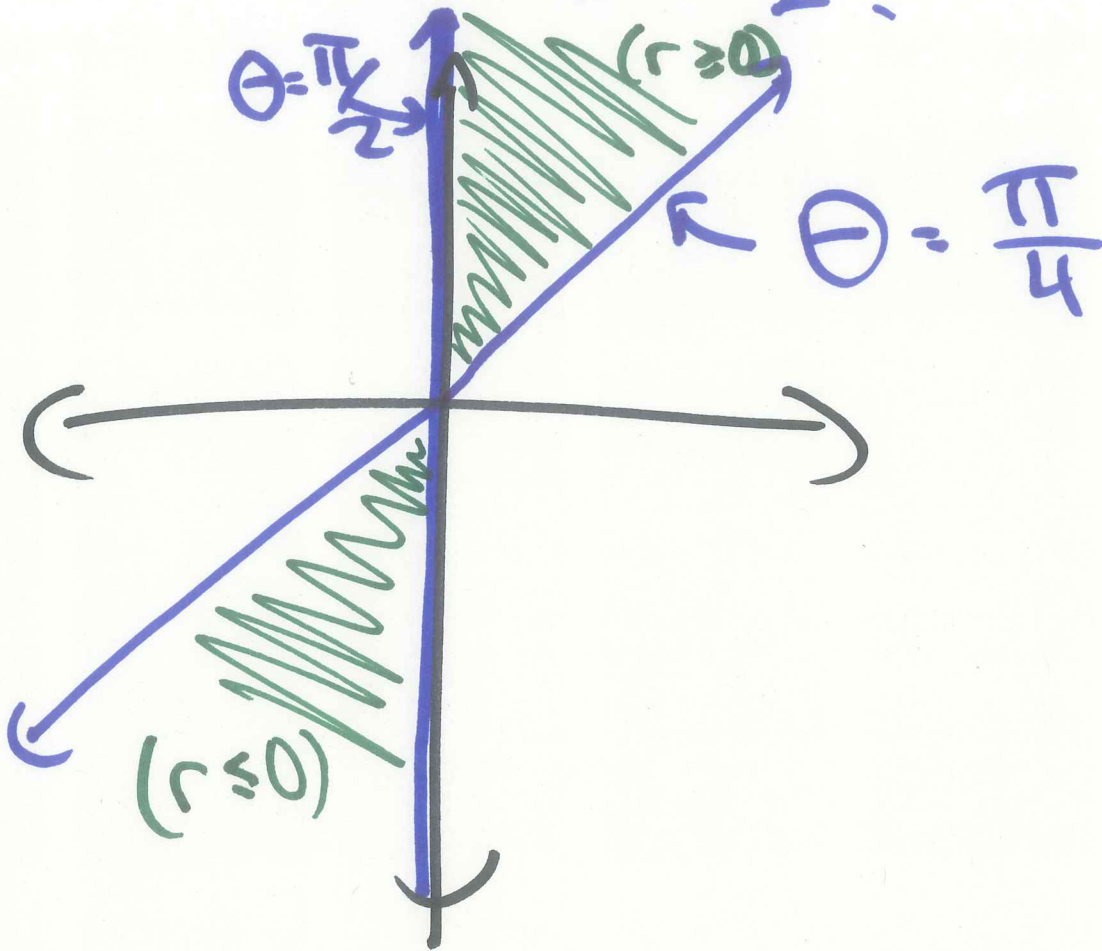




Ex:  $1 \leq r \leq 2$



Ex:  $\frac{F}{4} \leq \theta \leq \frac{F}{2}$



# Relating Polar and Cartesian Coordinates

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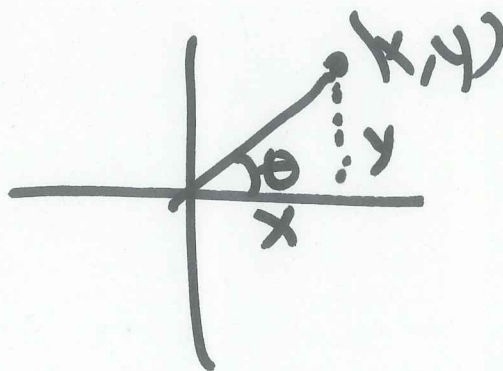
if  $(x, y) = P(r, \theta)$ , then

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$r^2 = x^2 + y^2$$

$$\tan \theta = \frac{y}{x}$$





Ex<sup>1</sup> Find a polar equation for the circle

$$(x-4)^2 + y^2 = 16$$

$$\Leftrightarrow x^2 - 8x + 16 + y^2 = 16$$

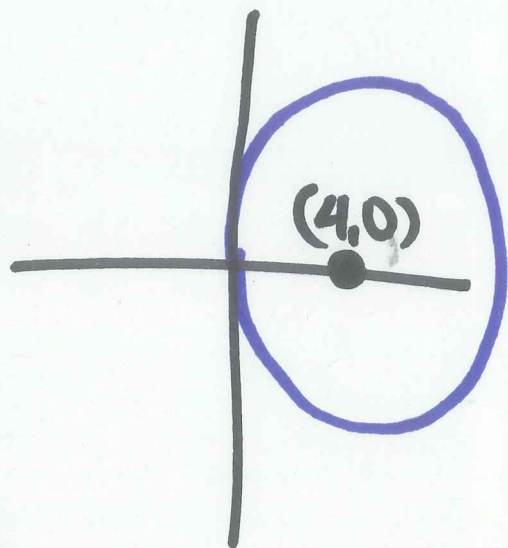
$$\Leftrightarrow (x^2 + y^2) - 8x = 0$$

$$\Leftrightarrow r^2 - 8r \cos \theta = 0$$

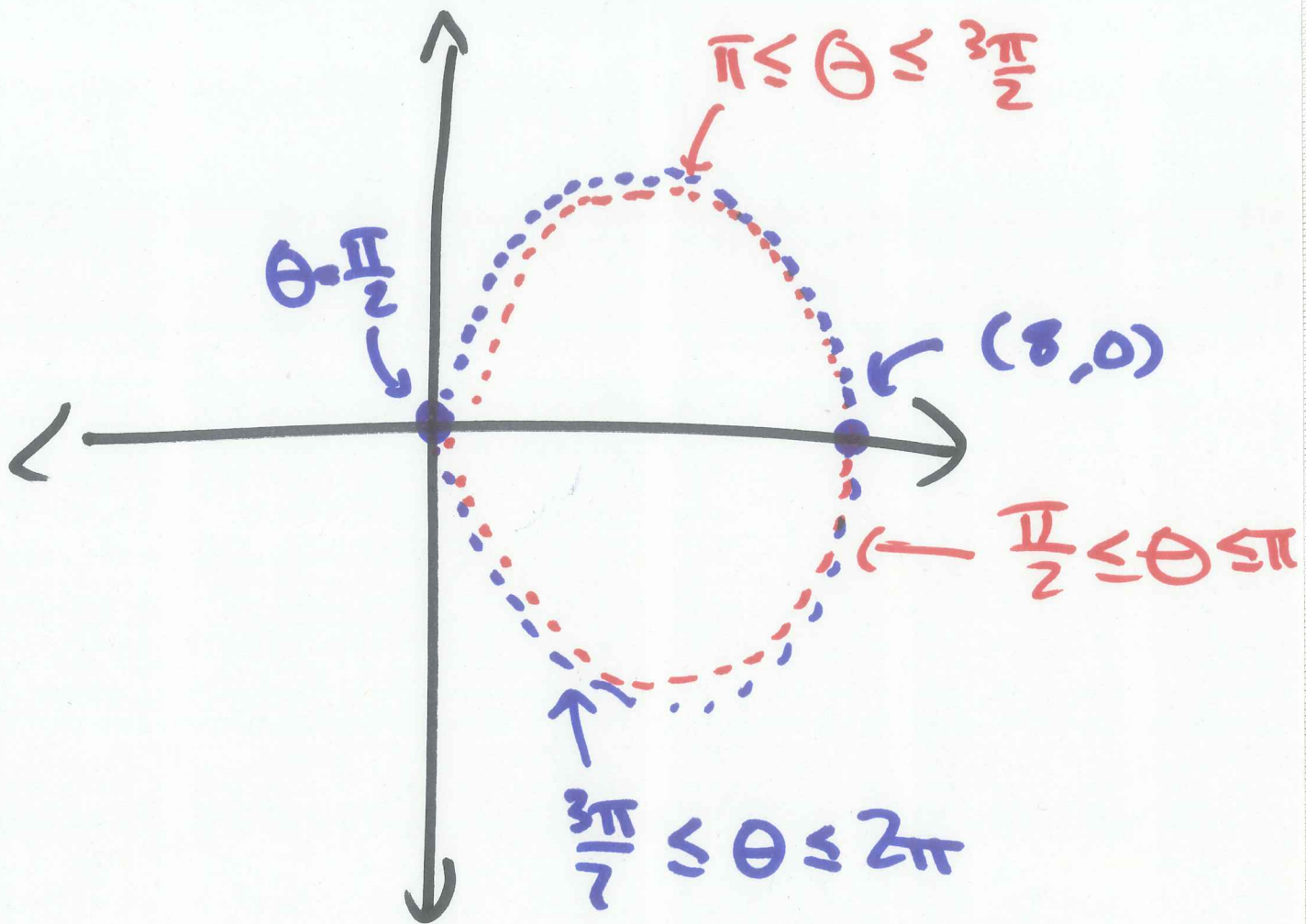
$$\Leftrightarrow r(r - 8 \cos \theta) = 0$$

$$\Leftrightarrow r = 0 \quad \underline{\text{or}} \quad r = 8 \cos \theta$$

$$\Leftrightarrow r = 8 \cos \theta \quad (\text{Same points})$$



$$r = 8 \cos \theta$$



Ex: Write a cartesian equation  
for the points  $P(r, \theta)$   
satisfying

$$r = \frac{5}{\sin \theta - 2 \cos \theta}$$

$$\Leftrightarrow r(\sin \theta - 2 \cos \theta) = 5$$

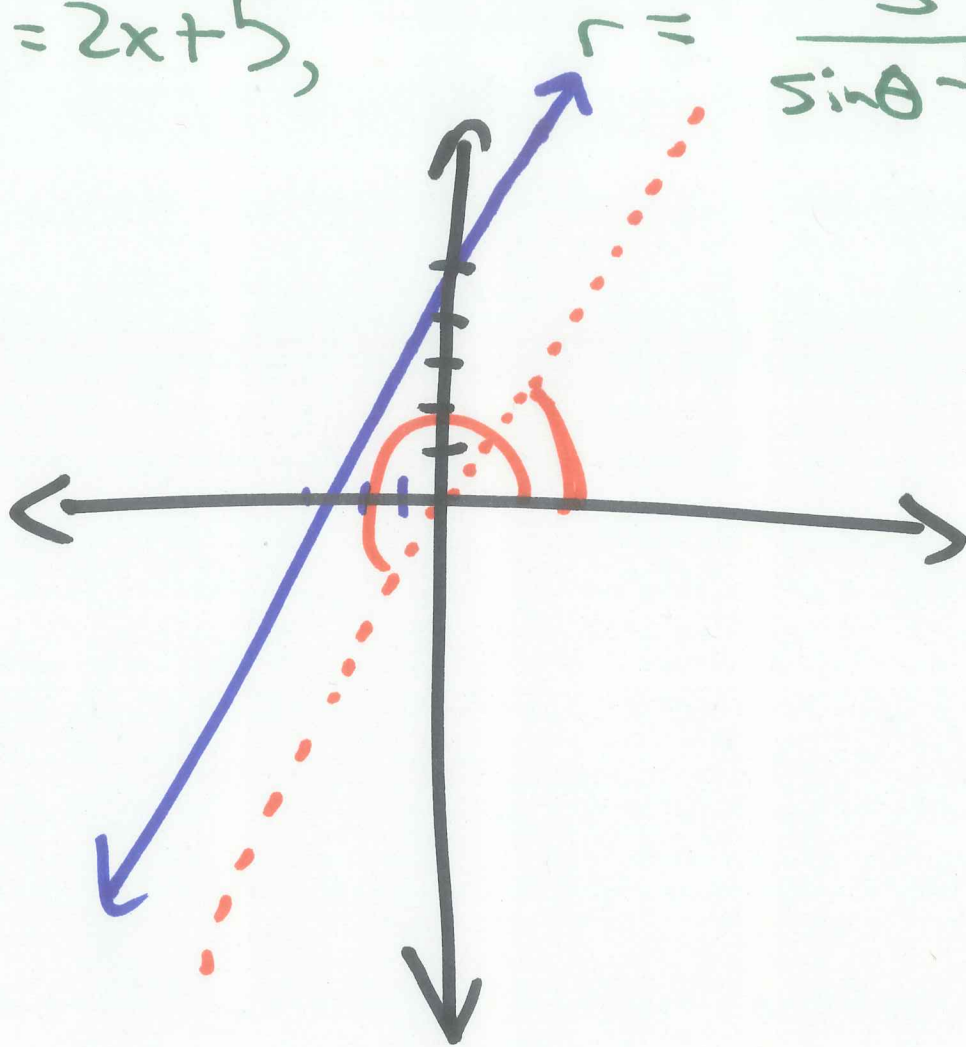
$$\Leftrightarrow \underline{r \sin \theta} - 2 \underline{r \cos \theta} = 5$$

$$\Leftrightarrow y - 2x = 5$$

$$\Leftrightarrow y = 2x + 5$$

$$y = 2x + 5,$$

$$r = \frac{5}{\sin\theta - 2\cos\theta}$$



$$\text{Ex: } r = \frac{5}{\sin\theta - 2\cos\theta}$$

$$\text{Ex: } r = \cot\theta \csc\theta$$

$$\text{Ex: } r = \csc\theta e^{r\cos\theta}$$

$$\text{Ex: } r = \theta \quad (\text{Draw})$$