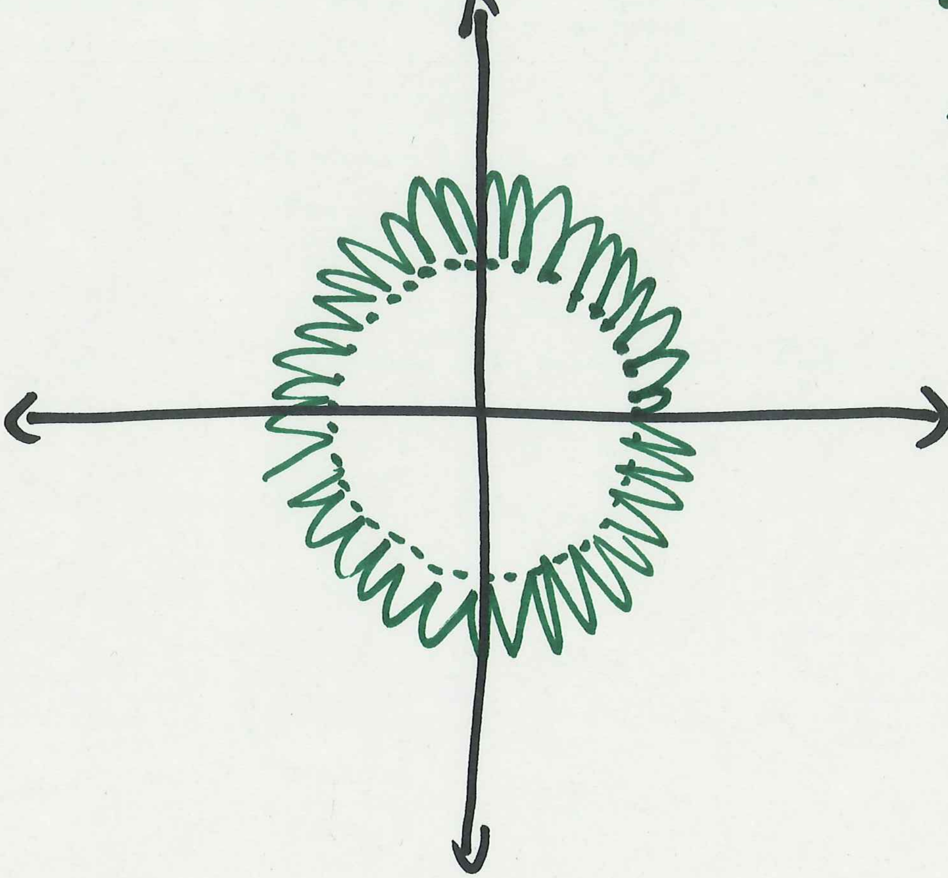
11.4 Graphing in Polar Coordinates

$$r = f(\theta)$$
$$= 1 + \sin^2(50\theta)$$



Symmetry:

Ex: $r = 3\cos 5\theta$.

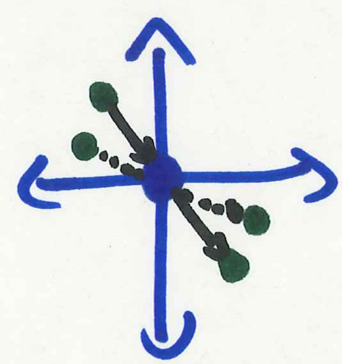
About x-axis:

If (r,θ) is in the curve, then so is $(r,-\theta)$ or $(-r, \pi-\theta)$.

About y-axis:

If (r,θ) is in the curve, then so is $(r, \pi-\theta)$ or $(-r,-\theta)$.

About Origin:



If (r,θ) is in the curve, then so is

$(-r,\theta)$ or $(r, \pi+\theta)$

If $r = \sin 2\theta$,

$(\sin 2\theta, \theta)$ in the curve.

Consider $\pi - \theta$.

$$(\sin(2(\pi-\theta)), \pi - \theta)$$

$$= (\sin(2\pi - 2\theta), \pi - \theta)$$

$$= (\sin(-2\theta), \pi - \theta)$$

$$= (-\sin 2\theta, \pi - \theta)$$

$$= (-r, \pi - \theta)$$

Slope of r Curve:

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

$$= \frac{\frac{d}{d\theta}\cdot(r\sin\theta)}{\frac{d}{d\theta}\cdot(r\cos\theta)}$$

$$= \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

$$= \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta} = \frac{dy}{dx}$$

(slope)

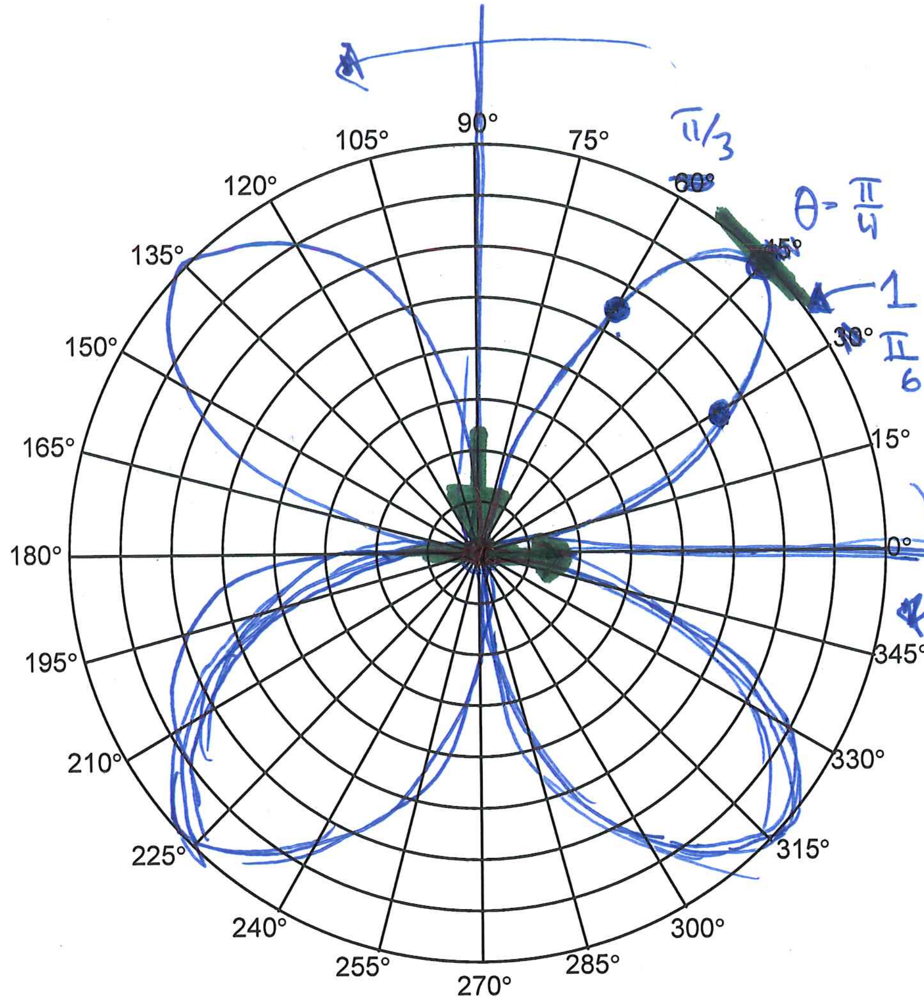
(when $r = f(\theta)$)

$$r = \sin 2\theta = f(\theta)$$

$$\frac{dr}{d\theta} = f'(\theta) = 2\cos 2\theta$$

$$\frac{dy}{dx} = \frac{2\cos 2\theta \cdot \sin\theta + \sin 2\theta \cos\theta}{2\cos 2\theta \cdot \cos\theta - \sin 2\theta \sin\theta}$$

θ	r	$\frac{dy}{dx}$
0	0	$\frac{2 \cdot \cos 0 \cdot \sin 0 + \sin 0 \cdot \cos 0}{} = 0$
$\frac{\pi}{2}$	0	$\frac{2\cos\pi \cdot \sin\frac{\pi}{2} + \sin\pi \cdot \cos\frac{\pi}{2}}{2\cos\pi \cdot \cos\frac{\pi}{2} - \sin\pi \cdot \sin\frac{\pi}{2}} = \text{DNE}$
$\frac{\pi}{4}$	$\sin\frac{\pi}{2} = 1$	$\frac{2\cos\frac{\pi}{2} \cdot \sin\frac{\pi}{4} + \sin\frac{\pi}{2}\cos\frac{\pi}{4}}{2\cos\frac{\pi}{2} \cdot \cos\frac{\pi}{4} - \sin\frac{\pi}{2}\sin\frac{\pi}{4}} = \frac{\sqrt{2}/2}{-\sqrt{2}/2} = -1$
$\frac{\pi}{6}$	$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$	



$$r = \underbrace{\sin 2\theta}$$

Period: π

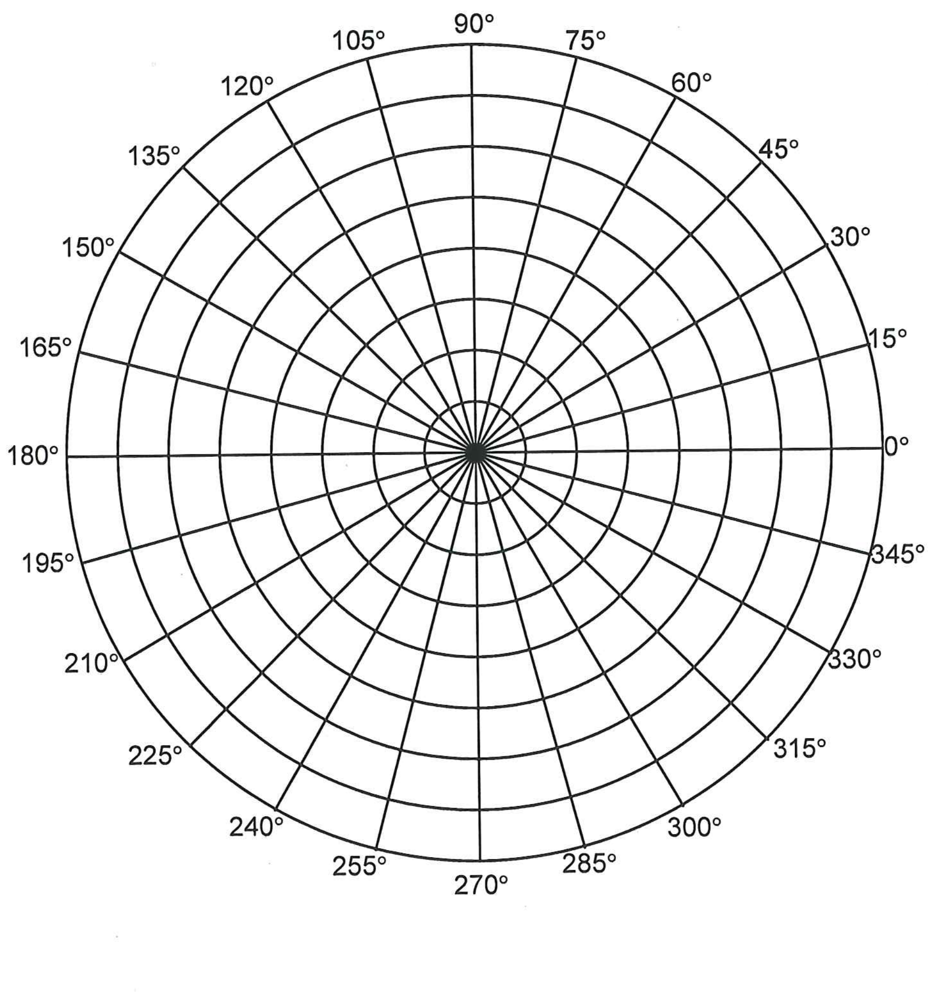
1 → Symmetry About Origin!

$(\overbrace{\sin 2\theta}^{r}, \theta)$

$(\sin(2\theta+2\pi), \theta+\pi)$

$$(\sin(2\theta+2\pi), \theta+\pi)$$

$$= (\underbrace{\sin(2\theta)}_{r}, \theta+\pi)$$



$$r^2 = 9 \sin\theta$$