COM S 330 — Functions and Compositions

Let A, B, and C be sets (not necessarily the same, but also not necessarily different), and let $f : A \to B$ and $g : B \to C$ be functions. Recall the definitions of injective, surjective, and bijective.

$f : A \to B$ injective (one-to-one)	$f : A \to B$ surjective (onto)	$f : A \to B$ bijective (one-to-one and onto)
$\forall x, y \in A[(f(x) = f(y)) \to (x = y)]$	$\forall y \in B \exists x \in A(f(x) = y)$	f is injective and surjective.
If $f(x) = f(y)$ for any two values x, y in the domain A, then x and y must be equal.	If y is in the codomain B, then there exists an element x from the domain A such that $f(x) = y$.	There exists an inverse function $f^{-1} : B \to A$ such that $f^{-1}(y) = x$ if and only if $f(x) = y$.

The *composition* function $g \circ f : A \to C$ is defined by $(g \circ f)(x) = g(f(x))$ for all $x \in A$. In the table below, we have specified different options for f and g being injective, surjective, neither, or both. You should determine if the given situations imply that $g \circ f$ is injective, surjective, neither, or both. Is it ALWAYS one of these cases, or could it possibly change for different examples? If your answers is "Always Yes" or "Always No" then give a proof. If your answer is "Sometimes Yes, Sometimes No" then give an example for each situation.

	g bijective	g injective, not surjective	g surjective, not injective	g not injective or surjective
f bijective	Is $g \circ f$ injective? _____	Is $g \circ f$ injective? _____	Is $g \circ f$ injective? _____	Is $g \circ f$ injective? _____
	Is $g \circ f$ surjective? _____	Is $g \circ f$ surjective? _____	Is $g \circ f$ surjective? _____	Is $g \circ f$ surjective? _____
f injective, not surjective	Is $g \circ f$ injective? _____	Is $g \circ f$ injective? _____	Is $g \circ f$ injective? _____	Is $g \circ f$ injective? _____
	Is $g \circ f$ surjective? _____	Is $g \circ f$ surjective? _____	Is $g \circ f$ surjective? _____	Is $g \circ f$ surjective? _____
f surjective, not injective	Is $g \circ f$ injective? _____	Is $g \circ f$ injective? _____	Is $g \circ f$ injective? _____	Is $g \circ f$ injective? _____
	Is $g \circ f$ surjective? _____	Is $g \circ f$ surjective? _____	Is $g \circ f$ surjective? _____	Is $g \circ f$ surjective? _____
f not injective or surjective	Is $g \circ f$ injective? _____	Is $g \circ f$ injective? _____	Is $g \circ f$ injective? _____	Is $g \circ f$ injective? _____
	Is $g \circ f$ surjective? _____	Is $g \circ f$ surjective? _____	Is $g \circ f$ surjective? _____	Is $g \circ f$ surjective? _____