

Type your answers to the following questions and submit a PDF file to Blackboard. One page per problem.

Problem 1. [5pts] Construct a truth table for the compound proposition $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$.

Problem 2. [5pts] Construct a truth table for the compound propositions $((p \rightarrow (q \rightarrow r)) \rightarrow s)$.

Problem 3. [10pts] On the island of Flopi, there are three types of people: Knights, Knaves, and Floppers. All inhabitants know which type the others are, but they are otherwise indistinguishable. Knights always tell the truth. Knaves always lie. Floppers always choose to lie or tell the truth by doing the opposite of the previous speaker (i.e. if someone just spoke a lie, the flopper will tell the truth; if someone just spoke a truth, the flopper will lie). While on your vacation, you come across three inhabitants, A , B , and C . They say the following, in order:

A says, “We are all knights.”

B says, “ C is a knight.”

C says, “ A is a knave.”

A says, “ C lied.”

Determine all possibilities of A , B , and C being Knights, Knaves, or Floppers (not all need to be distinct).

Problem 4. [5pts] Show that $[p \wedge (p \rightarrow q)] \rightarrow q$ is a tautology using truth tables.

Problem 5. [10pts] Prove¹ that $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \vee r)$ are logically equivalent (without using this equivalence from the tables).

Problem 6. [5pts] Find an assignment of the variables p, q, r such that the proposition

$$(p \vee \neg q) \wedge (p \vee q) \wedge (q \vee r) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \wedge (r \vee p)$$

is satisfied. For a bonus 5 points, prove that this assignment is *unique*. (Hint: Prove an equivalence between this proposition and the proposition $(p \leftrightarrow X) \wedge (q \leftrightarrow Y) \wedge (r \leftrightarrow Z)$ where (X, Y, Z) is your assignment.)

¹“Prove” means do not use truth tables!