Type your answers to the following questions and submit a PDF file to Blackboard. One page per problem.

Problem 1. [5pts] Let $P(x)$ be the statement "$\frac{1}{x} = x$." If the domain consists of real numbers, then what are these truth values?

a. $P(2)$.

False. $\frac{1}{2} \neq 2$.

b. $P(1)$.

True. $\frac{1}{1} = 1$.

c. $\exists x P(x)$.

True, since it is true for $x = 1$.

d. $\forall x \neq 0 (P(x))$.

False. $x = 2$ is a counterexample.

Problem 2. [5pts] Let $P(x, y)$ be the statement "$x > y$ and x is divisible by y." Let the domain consist of positive integers and write English sentences describing the following propositions.

a. $\exists x \forall y P(x, y)$.

There exists a positive integer x such that for each positive integer y, $x > y$ and x is divisible by y.

b. $\exists y \forall x P(x, y)$

There exists a positive integer y such that for each positive integer x, $x > y$ and x is divisible by y.

c. $\forall x \exists y P(x, y)$

For every positive integer x, there exists a positive integer y such that $x > y$ and x is divisible by y.

d. $\forall y \exists x P(x, y)$

For every positive integer y, there exists a positive integer x such that $x > y$ and x is divisible by y.

Problem 3. [10pts] Let $P(x, y)$, $Q(x, y)$, and $R(x)$ be propositional functions. Use logical equivalences to show that $\neg\forall x((\exists y(P(x, y) \to Q(x, y))) \vee R(x))$ and $\exists x(\neg R(x) \wedge \forall y(\neg Q(x, y) \wedge P(x, y)))$ are equivalent.

Solution:

$\neg\forall x((\exists y(P(x, y) \to Q(x, y))) \vee R(x))$	
$\equiv \exists x\neg((\exists y(P(x, y) \to Q(x, y))) \vee R(x))$	DeMorgan's Law (for Quantifiers)
$\equiv \exists x(\neg(\exists y(P(x, y) \to Q(x, y))) \wedge \neg R(x))$	DeMorgan's Law
$\equiv \exists x((\forall y\neg(P(x, y) \to Q(x, y))) \wedge \neg R(x))$	DeMorgan's Law (for Quantifiers)
$\equiv \exists x((\forall y\neg(\neg P(x, y) \vee Q(x, y))) \wedge \neg R(x))$	Logical Equivalence using Conditionals
$\equiv \exists x((\forall y(\neg(\neg P(x, y)) \wedge \neg Q(x, y))) \wedge \neg R(x))$	DeMorgan's Law
$\equiv \exists x((\forall y(P(x, y) \wedge \neg Q(x, y))) \wedge \neg R(x))$	Double Negation Law
$\equiv \exists x(\neg R(x) \wedge (\forall y(\neg Q(x, y) \wedge P(x, y))))$	Commutative Law

Problem 4. [10pts] Use rules of inference to show that if $p \wedge q$, $r \vee s$, and $p \to \neg r$, then s is true.

Solution:

Step		Reason
1.	$r \vee s$	Premise
2.	$\neg r \to s$	Logical Equivalence using Conditionals from (1).
3.	$p \to \neg r$	Premise.
4.	$p \to s$	Hypothetical Syllogism from (2) and (3).
5.	$p \wedge q$	Premise.
6.	p	Simplification from (5).
$\therefore$	s	Modus ponens from (4) and (6).

Problem 5. [10pts] Use rules of inference to show that if $(p \to q) \land (q \to p)$, $t \lor q$, $t \lor p$, and $(p \land q) \to t$, then t is true.

Solution: Note that there are many ways to solve this problem, as only two of the first three premises are required. Any pair of those three (with the fourth) suffice to demonstrate the statement.

Step		**Reason**
1.	$t \lor p$	Premise
2.	$t \lor q$	Premise
3.	$(t \lor p) \land (t \lor q)$	Conjunction from (1) and (2).
4.	$t \lor (q \land p)$	Distributive Law from (3).
5.	$\neg(q \land p) \to t$	Logical equivalence using conditionals from (4).
6.	$(p \land q) \to t$	Premise.
7.	$[(p \land q) \to t] \land [\neg(p \land q) \to t]$	Conjunction from (5) and (6).
8.	$[(p \land q) \lor \neg(p \land q)] \to t$	Logical equivalence using conditionals from (7).
9.	$(p \land q) \lor \neg(p \land q)$	Negation law.
$\therefore$	t	Modus ponens from (8) and (9).