Type your answers to the following questions and submit a PDF file to Blackboard. One page per problem.

Problem 1. [5pts] Give a direct proof of the following statement: If x and y are rational numbers, then $x + y$ and xy are rational numbers.

Problem 2. [5pts] Give a proof of the following statement: If x is a rational number and y is irrational, then $x + y$ and xy are irrational.

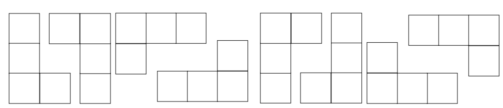
Problem 3. [5pts] Prove that $\sqrt{35}$ is irrational.

Problem 4. [10pts] In the country of Togliristan (where Knights, Knaves, and Togglers live), Togglers will alternate between telling the truth and lying (no matter what other people say). You meet two people, A and B. They say, in order:

A : B is a Knave.
B : A is a Knave.
A : B is a Knight.
B : A is a Toggler.

Determine what types of people A and B are.

Problem 5. [15pts] The *L-shaped Tetris piece* (or *tetromino*, see the Wikipedia page) consists of four squares: three of which are in a line and a fourth attached to one end of that line. See the figure below for all of the arrangements of the L-shaped Tetris piece (or L-piece).



Adapt the proofs from the book and class about domino tilings to prove the following:

a. [5pts] If m is an even number and n is a multiple of 4, then the $m \times n$ chessboard can be tiled using L-pieces.

b. [5pts] If a chessboard has a tilling using L-pieces, then the chessboard has a domino tiling.

c. [5pts] If m and n are odd numbers, then the $m \times n$ chessboard cannot be tiled using L-pieces.

d. [Bonus] Consider a $3 \times 4k$ chessboard.

[2pts] Show that if k is even, then the $3 \times 4k$ chessboard can be tiled using L-pieces.

[3pts] Show that if k is odd, then the $3 \times 4k$ chessboard *cannot* be tiled using L-pieces. (Hint: For the odd case, you may need the principle of *strong induction*. In short: Prove that you cannot tile a 3×4 chessboard, then assume that there is no tiling of a $3 \times 4\ell$ chessboard for all odd $\ell < k$, and use that to prove there is no tiling of a $3 \times 4k$ chessboard. THIS IS HARD.)