Type your answers to the following questions and submit a PDF file to Blackboard. One page per problem.

Problem 1. [10pts] Consider the following sequence definitions. Write out the first 10 terms ($n \in \{0, \dots, 9\}$) of each sequence.

a. [3pts] $a_0 = 2$ and $a_n = 2a_{n-1} - 1$ for $n \geq 1$.

b. [3pts] $b_0 = 0$ and $b_n = b_{n-1} + \frac{1}{2^n}$ for $n \geq 1$.

c. [4pts] $c_0 = 1$, $c_1 = 2$, and $c_n = 2(c_{n-1} - c_{n-2})$ for $n \geq 2$.

Problem 2. [5pts] Let $\{d_n\}_{n=0}^{\infty}$ be a sequence with the recurrence relation $d_n = 2d_{n-1} - d_{n-2}$ for $n \geq 2$. (We do not specify the terms d_0, d_1.) Use backward reasoning to motivate the closed form $d_n = nd_1 - (n-1)d_0$.

Problem 3. [10pts] Mathematical induction can be used to prove things that can be proven directly. In the two problems below, you will essentially prove the induction step of an induction proof.

a. [5pts] For $n \geq 0$, let $P(n)$ denote the sentence "The $2 \times n$ chessboard can be tiled using dominoes." Prove "$P(n) \to P(n+1)$."

b. [5pts] For $n \geq 0$, let $Q(n)$ denote the sentence "The $3 \times 2n$ chessboard can be tiled using dominoes." Prove "$Q(n) \to Q(n+1)$."

Problem 4. [10pts] Define a sequence $\{f_n\}_{n=0}^{\infty}$ as $f_0 = 1$ and for $n \geq 1$, $f_n = \frac{1}{1+f_{n-1}}$. Prove that for $n \geq 0$, $f_n = \frac{F_{n+1}}{F_{n+2}}$, where $\{F_n\}_{n=0}^{\infty}$ is the Fibonacci sequence.

Problem 5. [10pts] Let $\{A_n\}_{n=1}^{\infty}$ be an arbitrary sequence of non-empty sets. Recall the *n-fold Cartesian product* $\prod_{i=1}^{n} A_i$ is the set of n-tuples $(a_1, \dots, a_n)$ where each a_i is an element in A_i. The *iterated product* is the set sequence $\{B_n\}_{n=1}^{\infty}$ where $B_1 = A_1$, and for $n \geq 2$, $B_n = B_{n-1} \times A_n$. (Thus, $B_1 = A_1$, $B_2 = A_1 \times A_2$, $B_3 = (A_1 \times A_2) \times A_3$, etc.) Use induction to prove that for all $n \geq 2$, there is a bijection f_n from n-fold Cartesian product $\prod_{i=1}^{n} A_i$ to the iterated product $B_n = B_{n-1} \times A_n$. [Note: the n-fold Cartesian product $\prod_{i=1}^{n} A_i$ consists of n-tuples $(a_1, \dots, a_n)$, while the iterated product B_n consists of ordered pairs (b, a_n) where $b \in B_{n-1}$ and $a_n \in A_n$.]