Type your answers to the following questions and submit a PDF file to Blackboard. One page per problem.

Problem 1. [10pts] Let $S \subseteq \mathbb{R}$ be defined recursively using (Basis Step) $2015 \in S$, (Constructive Step) if $x, y \in S$, then $x - y \in S$ and $x^{|y|} \in S$. Prove that $S = \mathbb{Z}$. [Hint: To show $\mathbb{Z} \subseteq S$, prove that 0 and -1 are in S, and use these to construct any integer you want.]

Problem 2. [10pts] Let $S \subseteq \mathbb{R}$ be defined recursively using (Basis Step) $0 \in S$, (Constructive Step) If $x \in S$, then $x + 1 \in S$, $x + \pi \in S$, and $x + \sqrt{2} \in S$. Prove that $S = \{a + b\pi + c\sqrt{2} : a, b, c \in \mathbb{N}\}$.

Problem 3. [10pts] Count the following things. Give your method and state which rules you are using.

1. [2pts] A *palindrome* is a word that reads the same forwards and backwards. How many binary words of length n are palindromes?
2. [2pts] Let $A = \{1, \ldots, n\}$ for some integer $n \geq 1$, and let $B = \{1, 2\}$. Count the number of *surjective* functions $f : A \to B$.
3. [3pts] The class with n students, m TAs, and 1 instructor holds an election. They elect a president and a vice president, but the vice-president cannot "out-rank" the president (the instructor out-ranks the TAs and the TAs outrank the students). How many ways can the election complete?
4. [3pts] How many ways can we rearrange the symbols "abcd123" such that the letters appear in order, the numbers appear in order, but the letters and numbers can be mixed?

Problem 4. [10pts] Demonstrate $\binom{n}{k}\binom{n-k}{\ell} = \binom{n}{k+\ell}\binom{k+\ell}{\ell}$ by using two methods to count the number of ways to create two disjoint committees of size k and ℓ from a group of n people